On the optimality of spiral search

Elmar Langetepe
University of Bonn

Searching for a point in the plane

Searching for a point in the plane

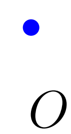
- Online search problem

Searching for a point in the plane

- Online search problem
- Searching in the plane

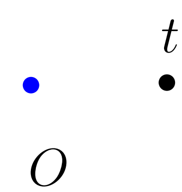
Searching for a point in the plane

- Online search problem
- Searching in the plane
- Startpoint O,



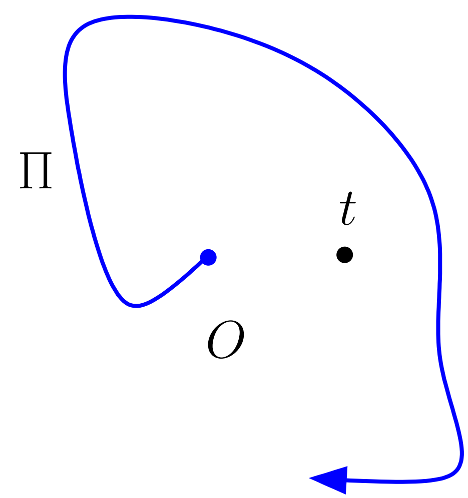
Searching for a point in the plane

- Online search problem
- Searching in the plane
- Startpoint O, unknown target t



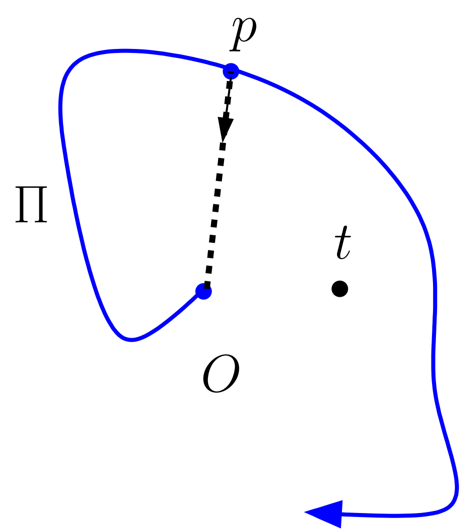
Searching for a point in the plane

- Online search problem
- Searching in the plane
- Startpoint O, unknown target t
- Strategy Π



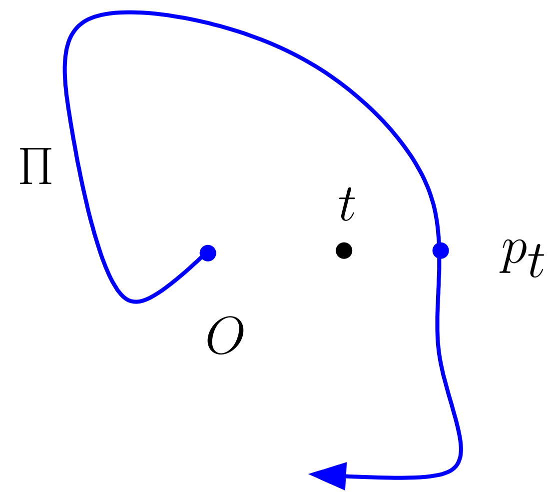
Searching for a point in the plane

- Online search problem
- Searching in the plane
- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane



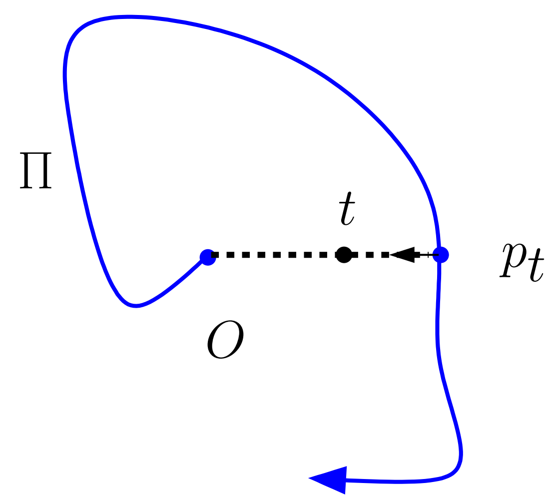
Searching for a point in the plane

- Online search problem
- Searching in the plane
- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane
- Find t if $t \in Op_t$



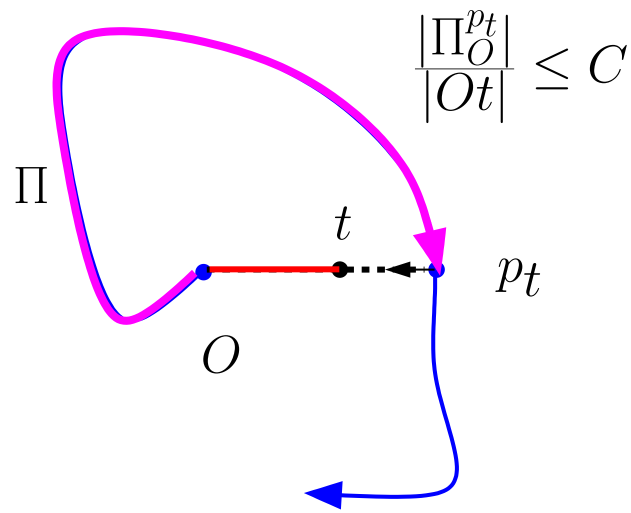
Searching for a point in the plane

- Online search problem
- Searching in the plane
- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane
- Find t if $t \in Op_t$



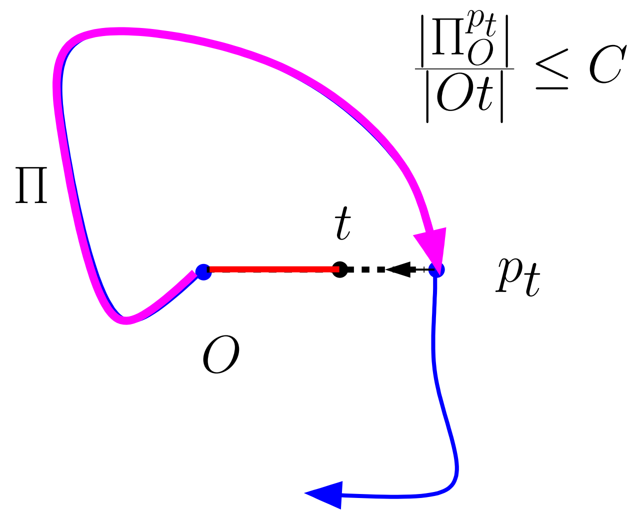
Searching for a point in the plane

- Online search problem
- Searching in the plane
- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane
- Find t if $t \in Op_t$
- Competitive ratio: worst-case
  $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$



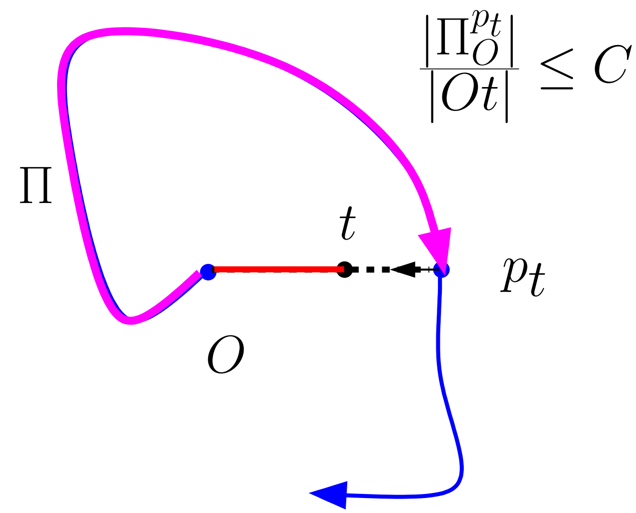
Searching for a point in the plane

- Online search problem
- Searching in the plane
- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane
- Find t if $t \in Op_t$
- Competitive ratio: worst-case $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$
- Shmuel Gal 1980!



Searching for a point in the plane

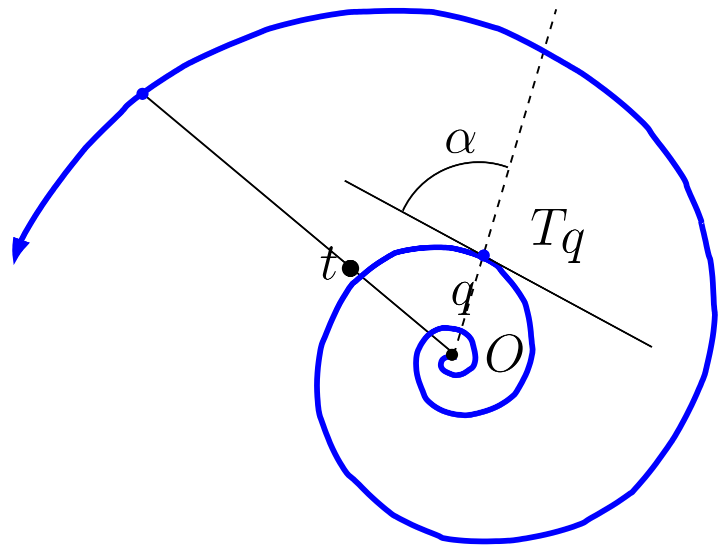
- Online search problem
- Searching in the plane
- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane
- Find t if $t \in Op_t$
- Competitive ratio: worst-case $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$
- Shmuel Gal 1980!
- Optimal search path?



Spiral search

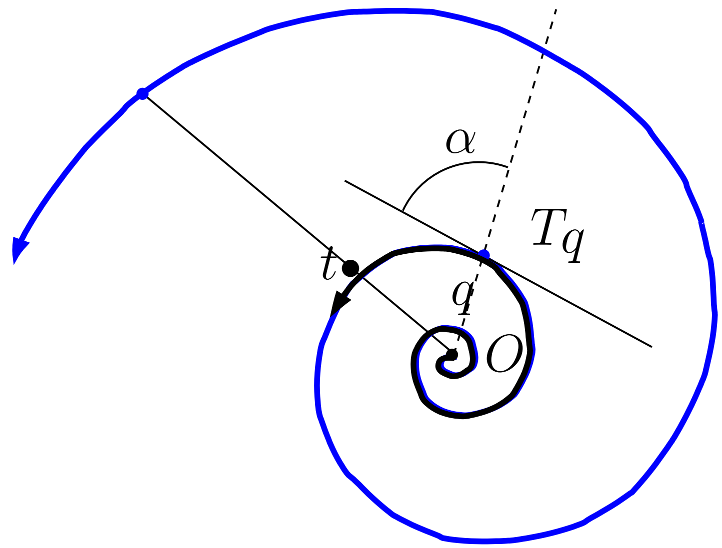
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$



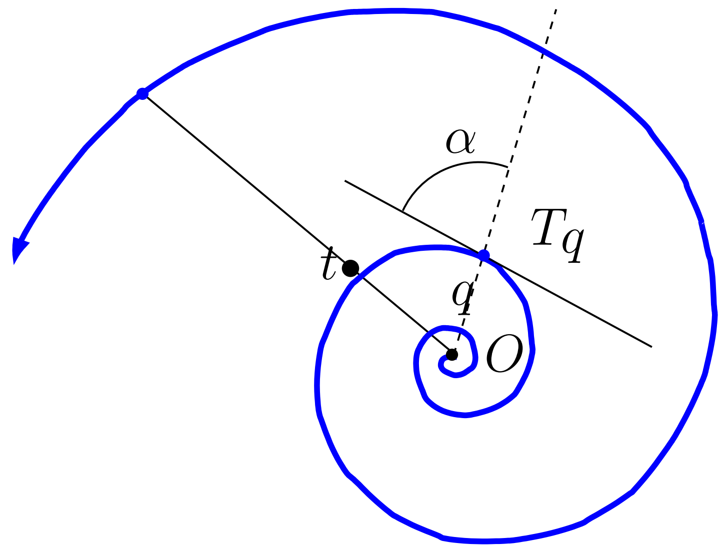
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target!



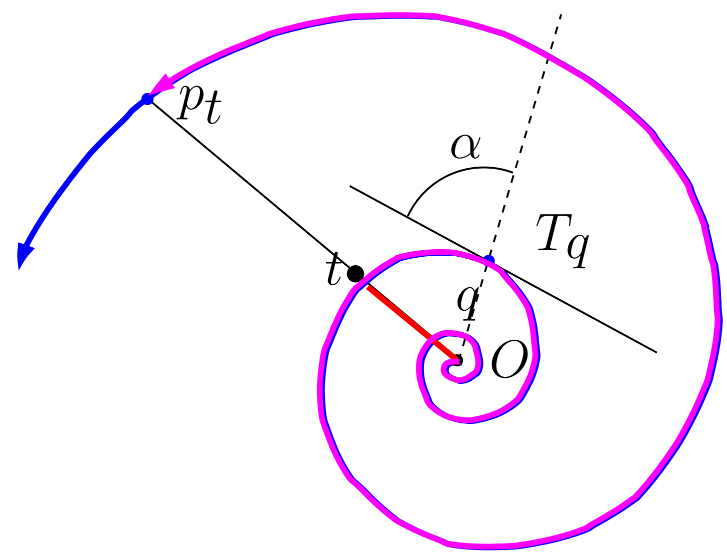
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target! Another full round!



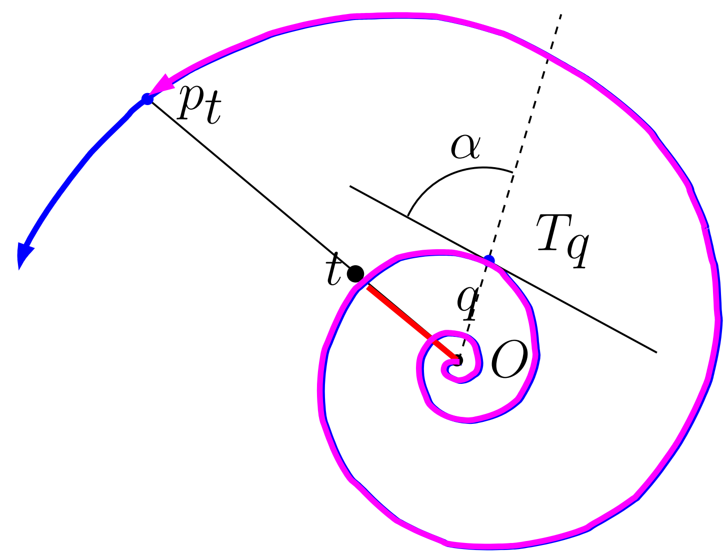
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target! Another full round!



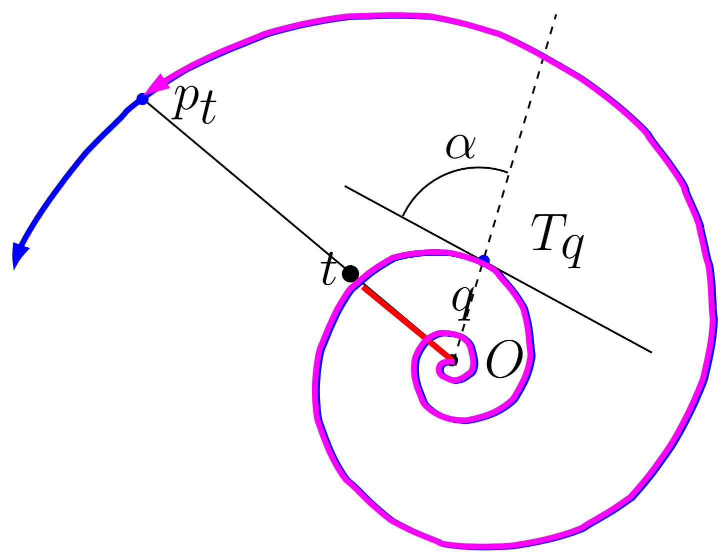
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target! Another full round!
- $C(\alpha) = \frac{|\Pi_O^{p_t}|}{|Ot|} = \frac{1}{\cos\alpha} e^{2\pi \cot \alpha}$



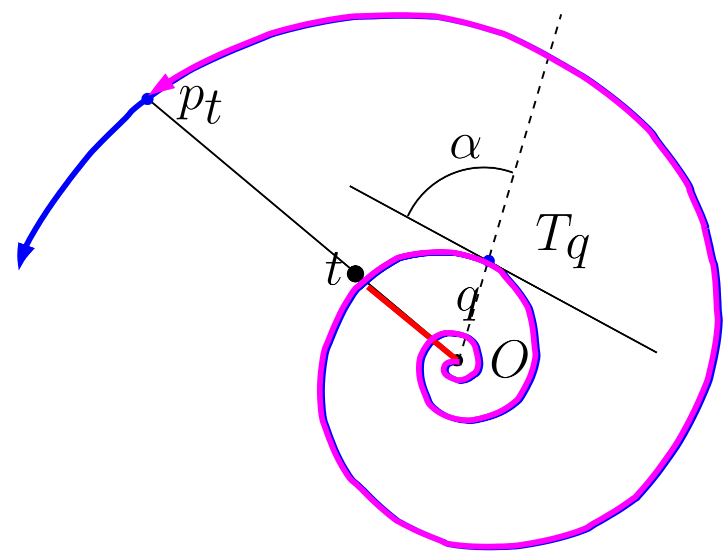
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target! Another full round!
- $C(\alpha) = \frac{|\Pi_O^{p_t}|}{|Ot|} = \frac{1}{\cos\alpha} e^{2\pi \cot\alpha}$
- $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$, $C = 17.289\ldots$



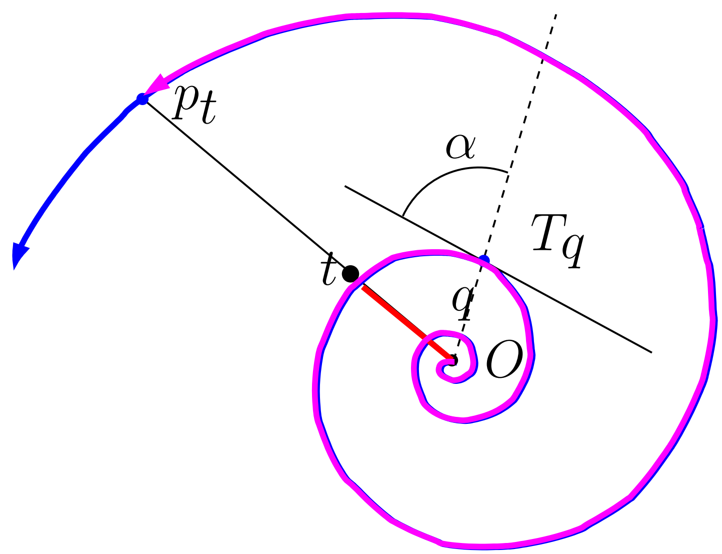
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target! Another full round!
- $C(\alpha) = \frac{|\Pi_O^{p_t}|}{|Ot|} = \frac{1}{\cos\alpha} e^{2\pi \cot \alpha}$
- $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$, $C = 17.289\ldots$
- Gal: Best periodic and monotone strategy!
  Is this optimal in general?



Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target! Another full round!
- $C(\alpha) = \frac{|\Pi_O^{p_t}|}{|Ot|} = \frac{1}{\cos \alpha} e^{2\pi \cot \alpha}$
- $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$, $C = 17.289\ldots$
- Gal: Best periodic and monotone strategy!
  Is this optimal in general?
- Many variants in the last decades: Spiral conjecture!



Technique for optimizing discrete problems

Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes*

$\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes* $\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Framework of Gal: Optimality of exponential function,

Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes* $\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Framework of Gal: Optimality of exponential function, conditions

Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes* $\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Framework of Gal: Optimality of exponential function, conditions

- Continuouity

Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes* $\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Framework of Gal: Optimality of exponential function, conditions

- Continuouity
- Unimodality:

Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes* $\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Framework of Gal: Optimality of exponential function, conditions

- Continuouity
- Unimodality: $F_k(S \cdot a) = F_k(a)$, $F_k(a+b) \leq \max\{F_k(a), F_k(b)\}$

Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes* $\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Framework of Gal: Optimality of exponential function, conditions

- Continuouity
- Unimodality: $F_k(S \cdot a) = F_k(a)$, $F_k(a+b) \leq \max\{F_k(a), F_k(b)\}$
- Other reasonable properties

Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes* $\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Framework of Gal: Optimality of exponential function, conditions

- Continuouity
- Unimodality: $F_k(S \cdot a) = F_k(a)$, $F_k(a+b) \leq \max\{F_k(a), F_k(b)\}$
- Other reasonable properties
- Conclusion: $a_i = d^i$ optimal

Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes* $\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Framework of Gal: Optimality of exponential function, conditions

- Continuouity
- Unimodality: $F_k(S \cdot a) = F_k(a)$, $F_k(a+b) \leq \max\{F_k(a), F_k(b)\}$
- Other reasonable properties
- Conclusion: $a_i = d^i$ optimal
- $\inf_d \frac{\sum_{i=1}^{k+1} d^i}{d^k} = 4$ for $d = 2$

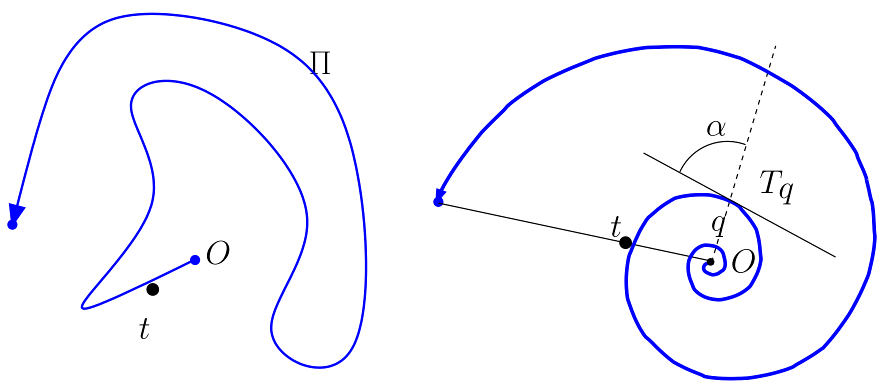
Technique for optimizing discrete problems

Find infinite sequence $a = (a_1, a_2, \ldots)$ that *minimizes* $\sup_k F_k(a)$ with $F_k(a) := \frac{\sum_{i=1}^{k+1} a_i}{a_k}$ (functionals F_k)

Framework of Gal: Optimality of exponential function, conditions

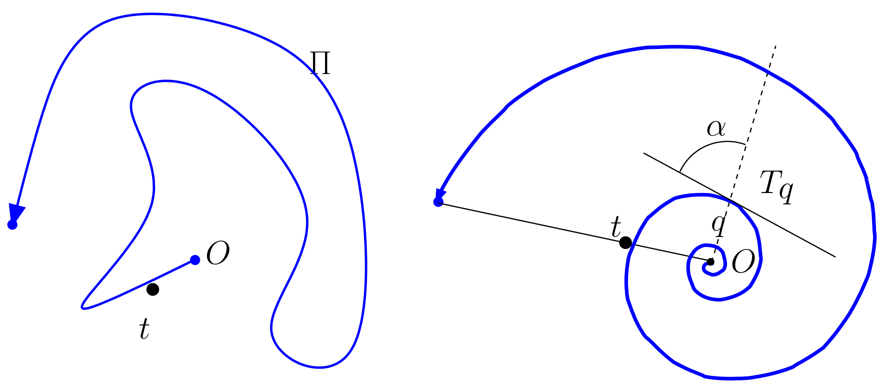
- Continuouity
- Unimodality: $F_k(S \cdot a) = F_k(a)$, $F_k(a+b) \leq \max\{F_k(a), F_k(b)\}$
- Other reasonable properties
- Conclusion: $a_i = d^i$ optimal
- $\inf_d \frac{\sum_{i=1}^{k+1} d^i}{d^k} = 4$ for $d = 2$
- Gal: Search Games 1980

Short summary



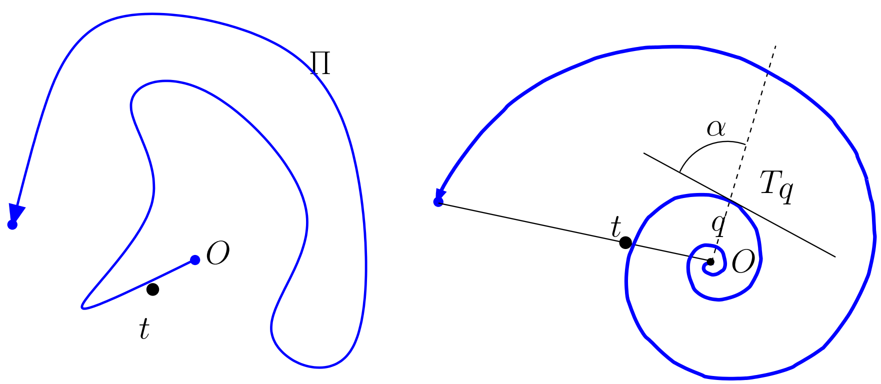
Short summary

- Old fundamental search game problem



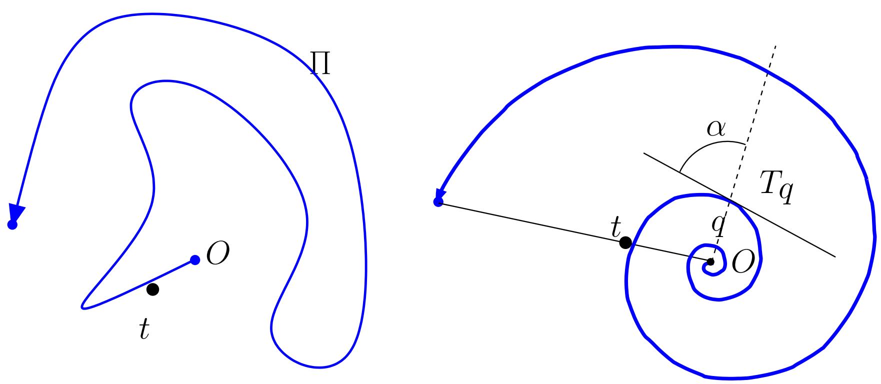
Short summary

- Old fundamental search game problem
- Framework of Gal for discrete scenarios



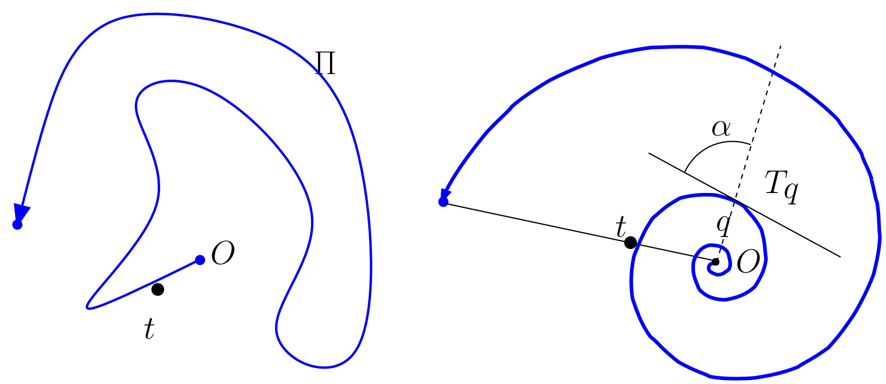
Short summary

- Old fundamental search game problem
- Framework of Gal for discrete scenarios
- Logarithmic spiral conjecture



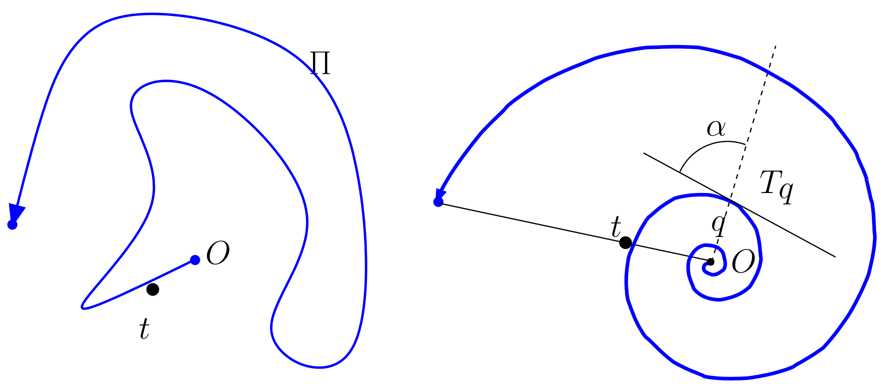
Short summary

- Old fundamental search game problem
- Framework of Gal for discrete scenarios
- Logarithmic spiral conjecture
- Find a discrete functional that gives a tight lower bound



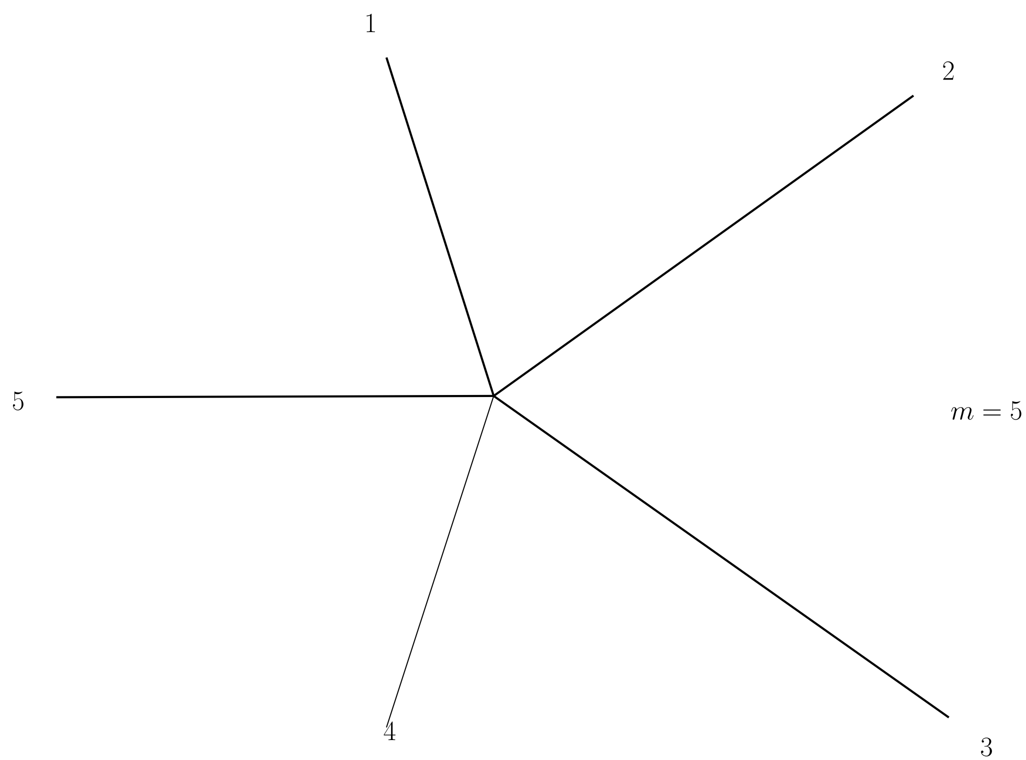
Short summary

- Old fundamental search game problem
- Framework of Gal for discrete scenarios
- Logarithmic spiral conjecture
- Find a discrete functional that gives a tight lower bound
- Ratio: $17.289\ldots$



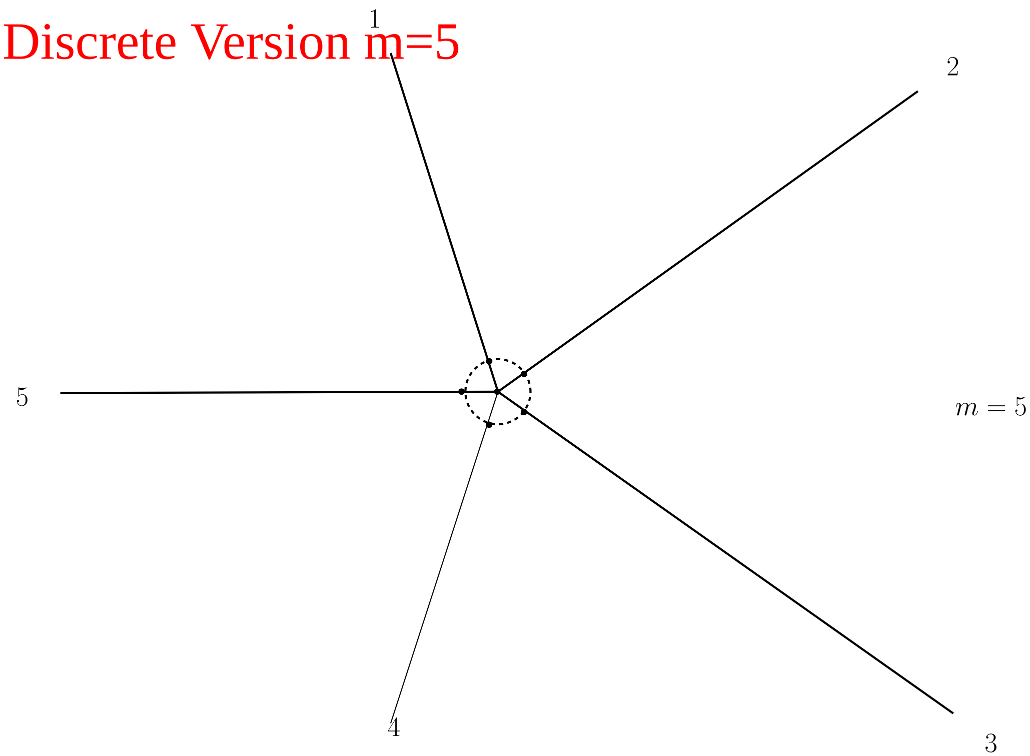
Lower bound construction: General overview

Lower bound construction: General overview

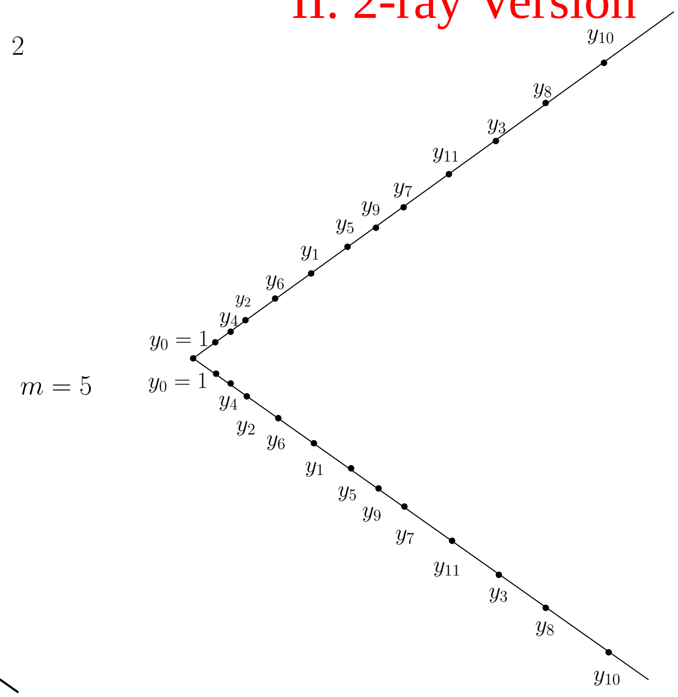


Lower bound construction: General overview

I. Discrete Version m=5

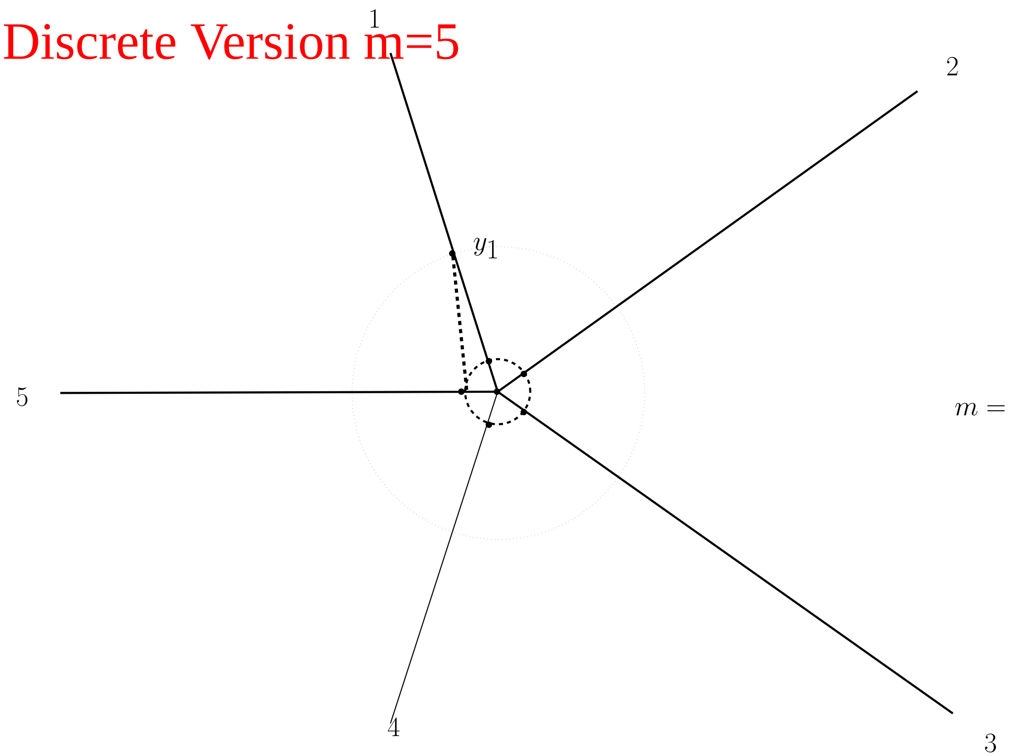


II. 2-ray Version

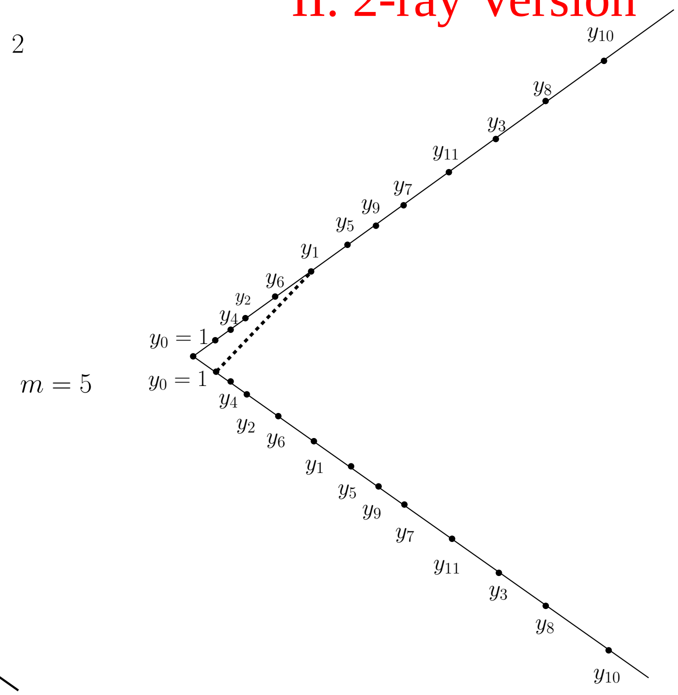


Lower bound construction: General overview

I. Discrete Version m=5

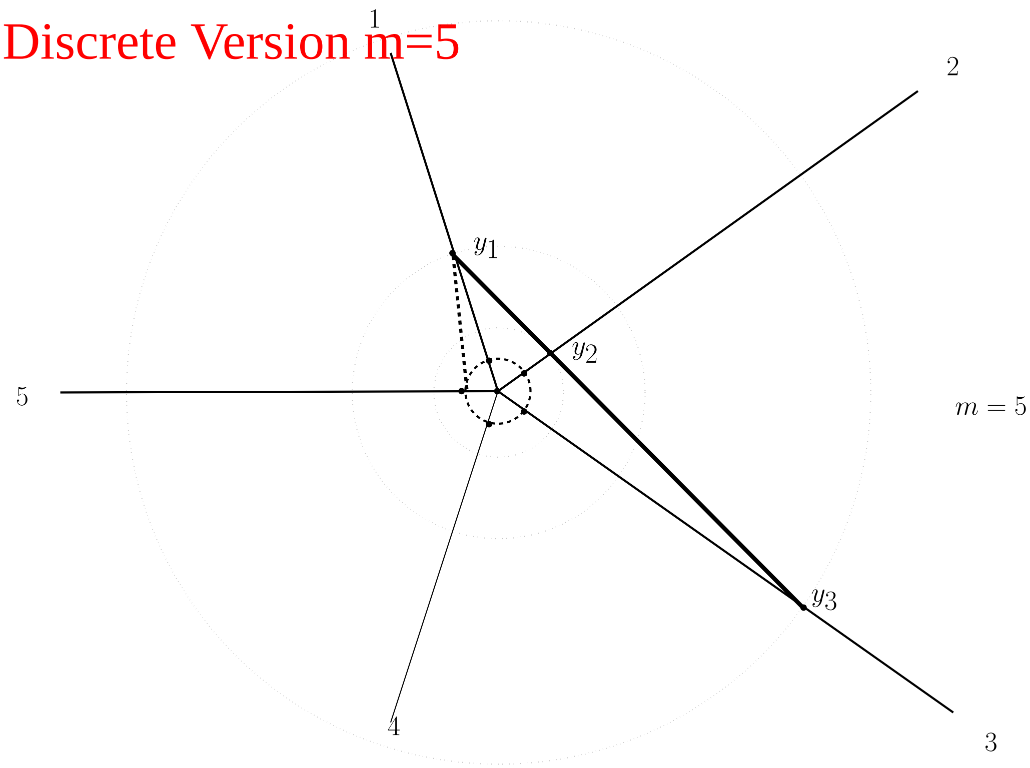


II. 2-ray Version

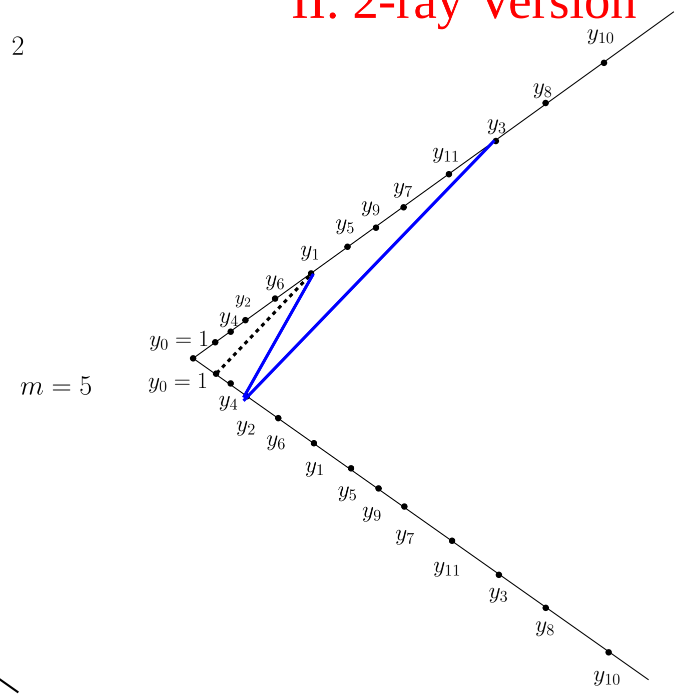


Lower bound construction: General overview

I. Discrete Version m=5

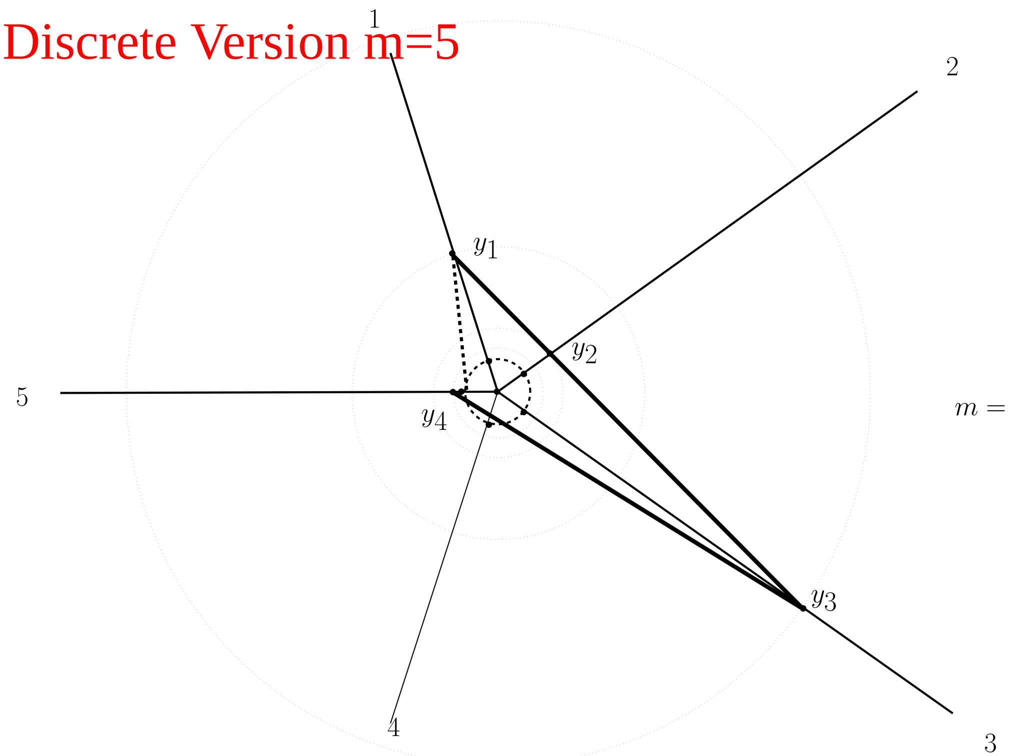


II. 2-ray Version

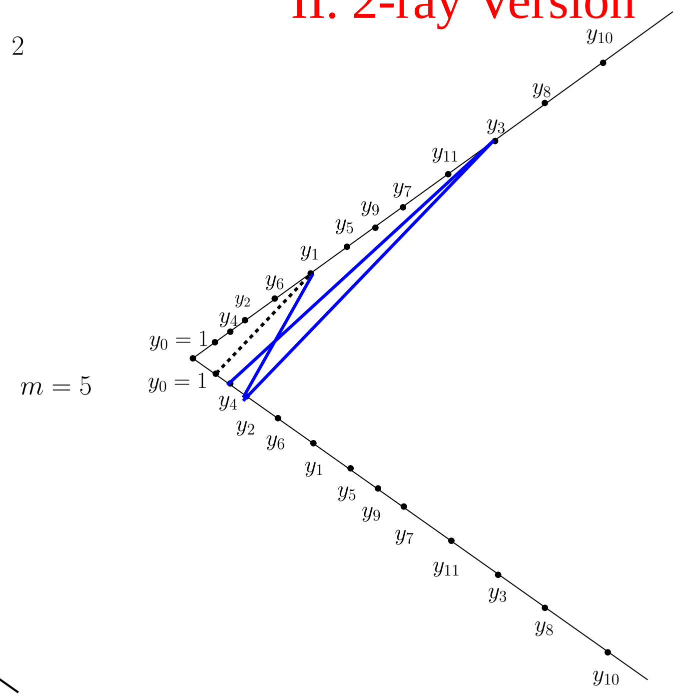


Lower bound construction: General overview

I. Discrete Version m=5

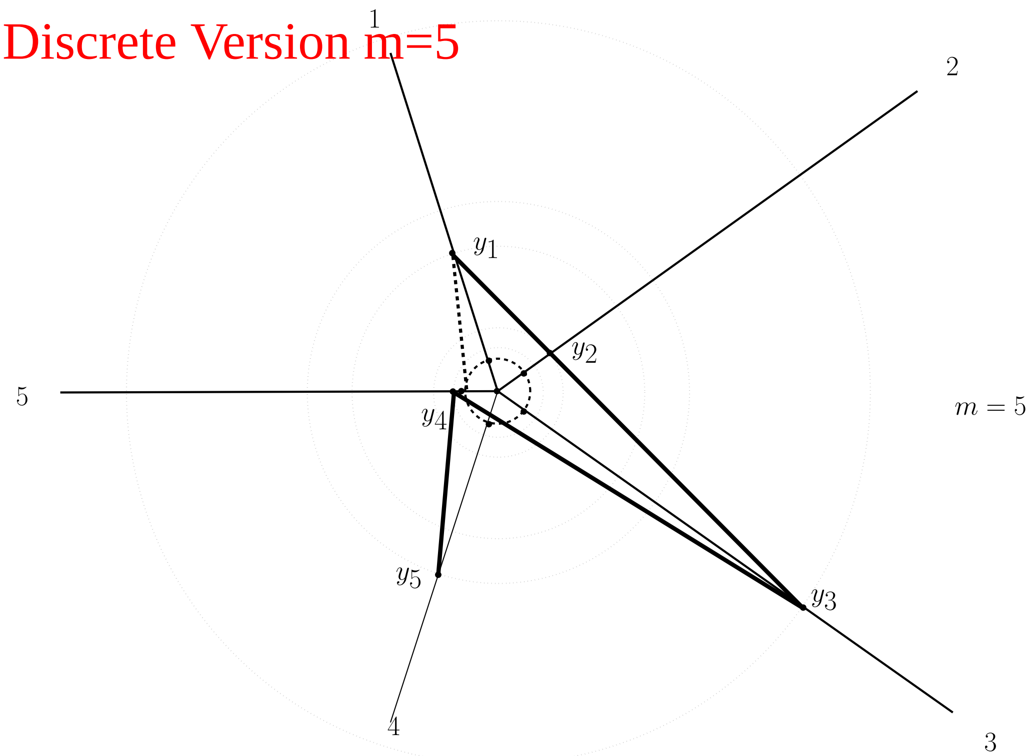


II. 2-ray Version

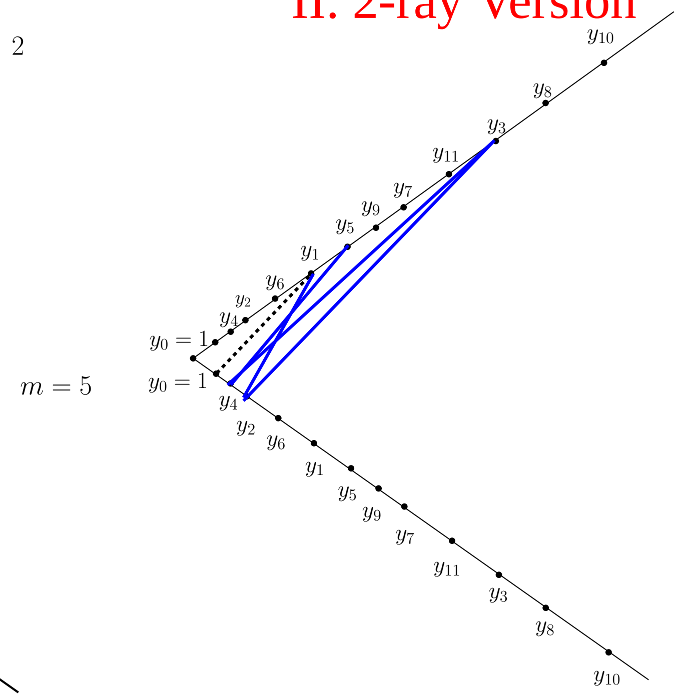


Lower bound construction: General overview

I. Discrete Version m=5

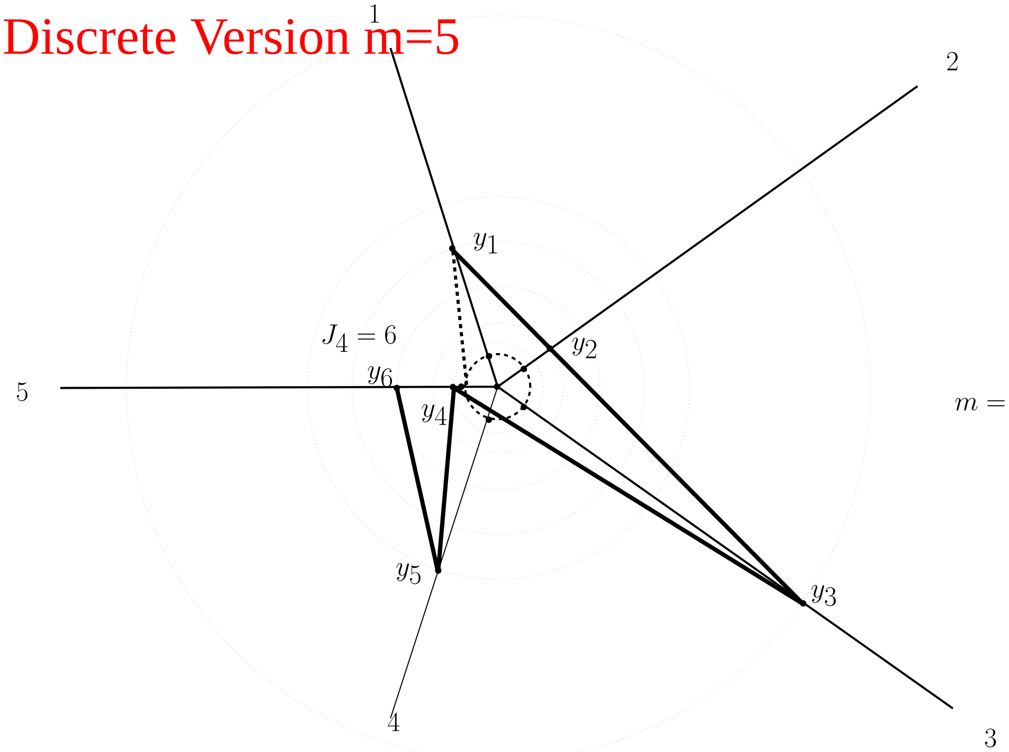


II. 2-ray Version

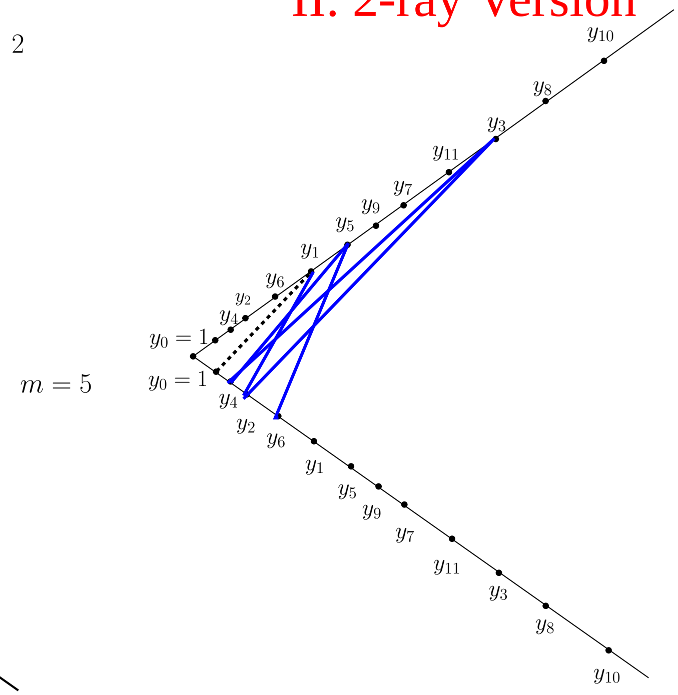


Lower bound construction: General overview

I. Discrete Version m=5

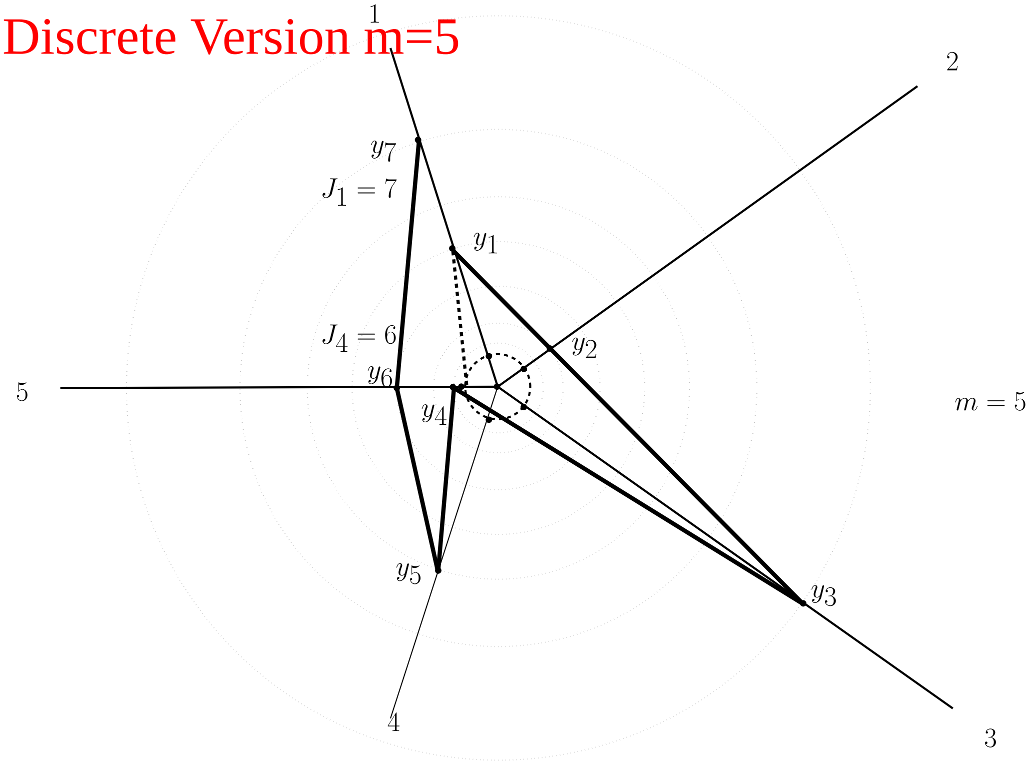


II. 2-ray Version

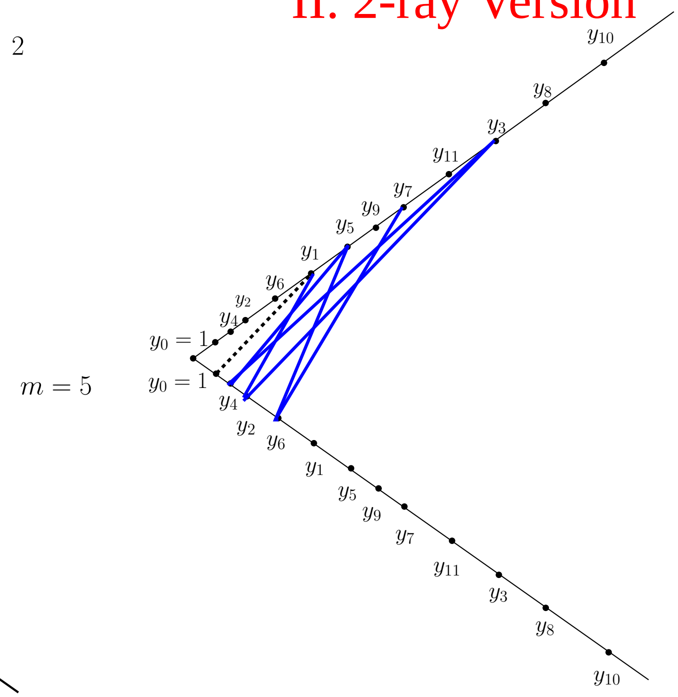


Lower bound construction: General overview

I. Discrete Version m=5

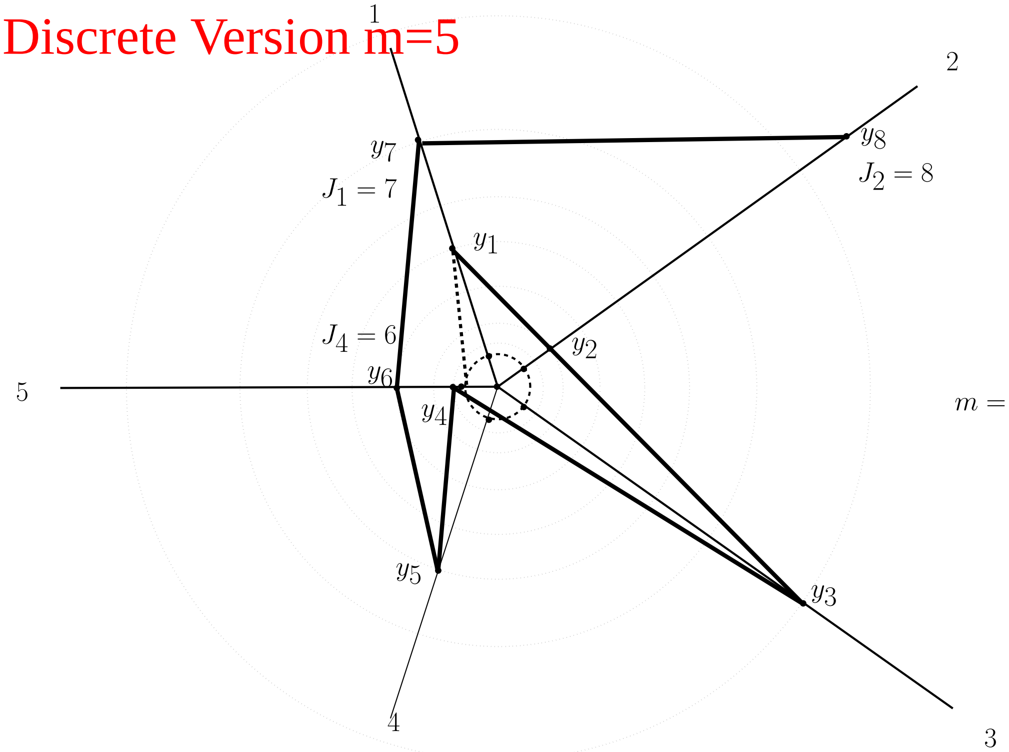


II. 2-ray Version

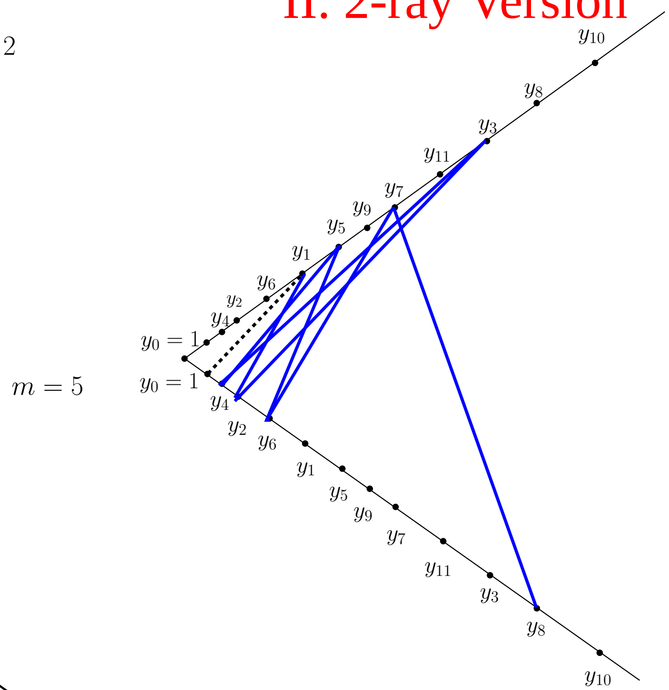


Lower bound construction: General overview

I. Discrete Version m=5

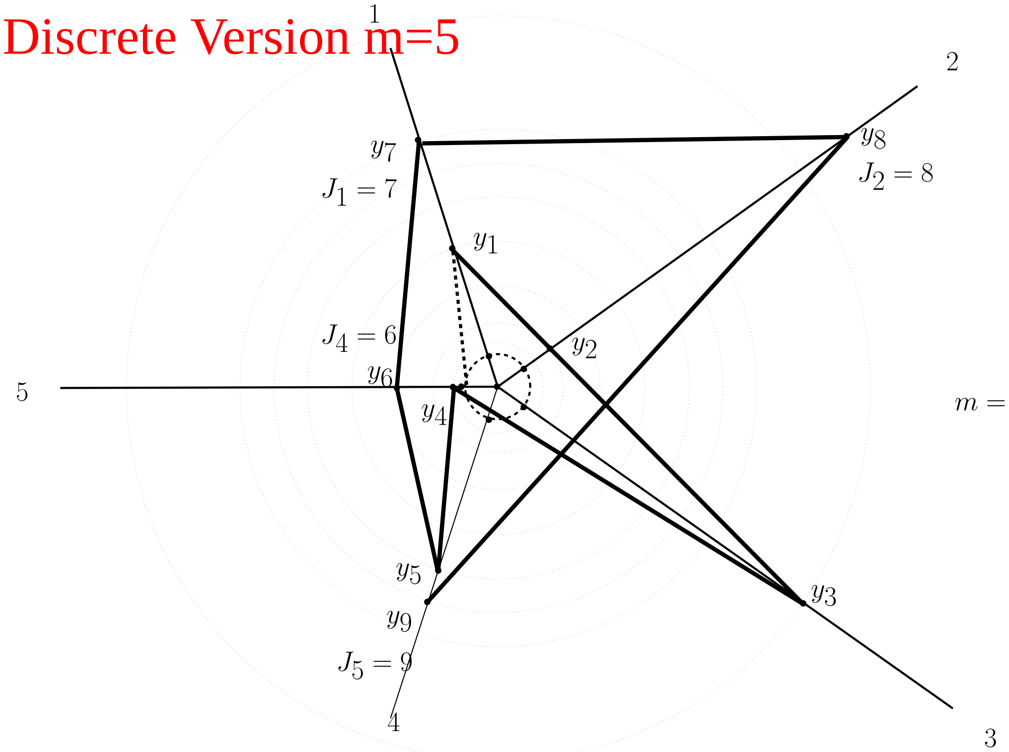


II. 2-ray Version

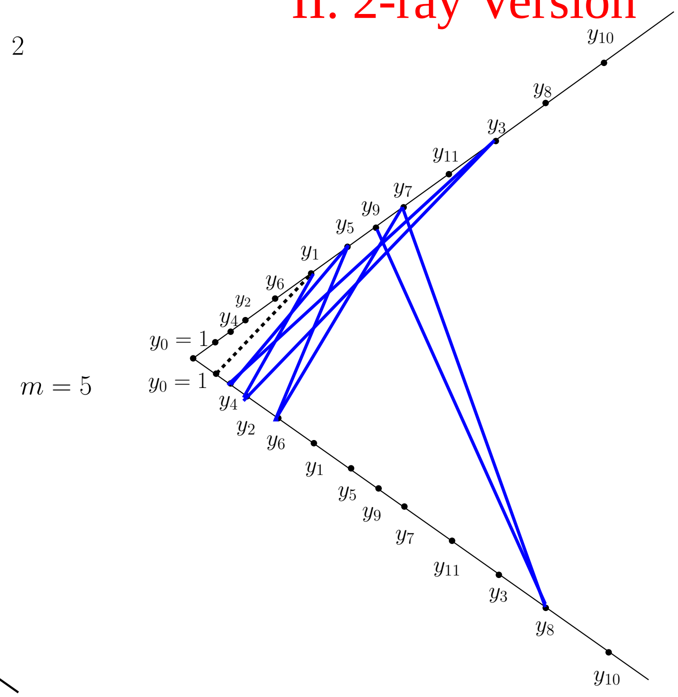


Lower bound construction: General overview

I. Discrete Version m=5

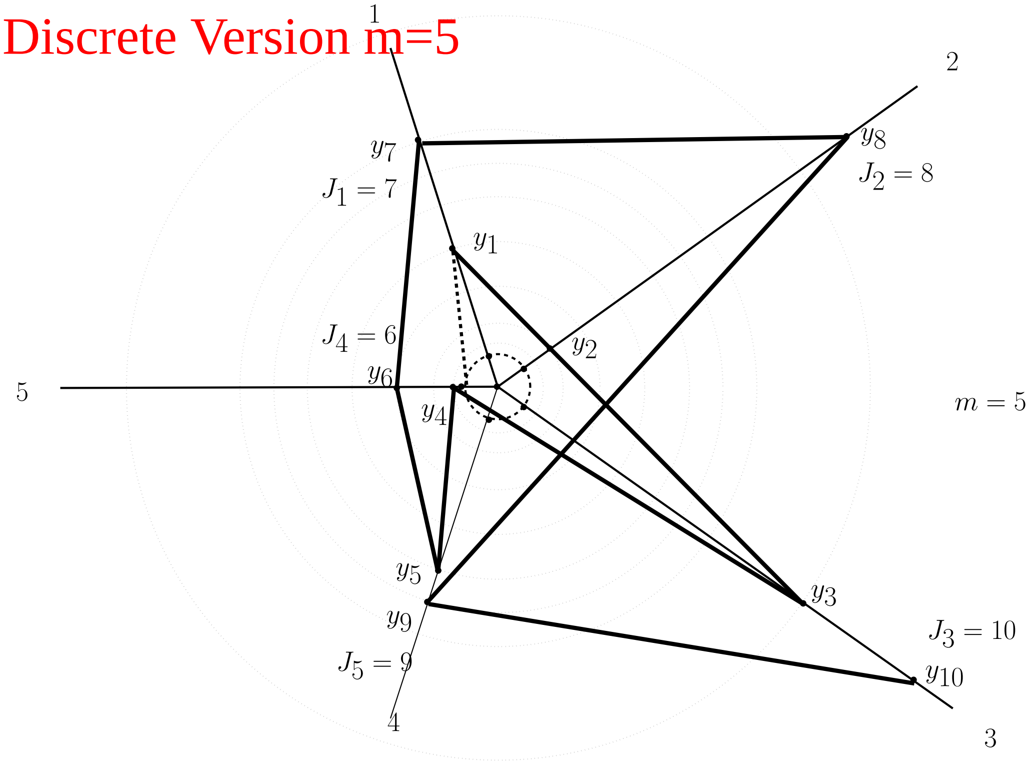


II. 2-ray Version

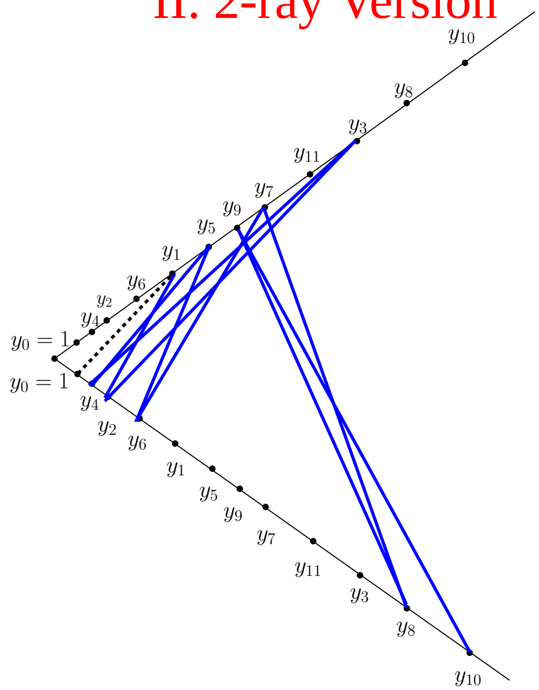


Lower bound construction: General overview

I. Discrete Version m=5

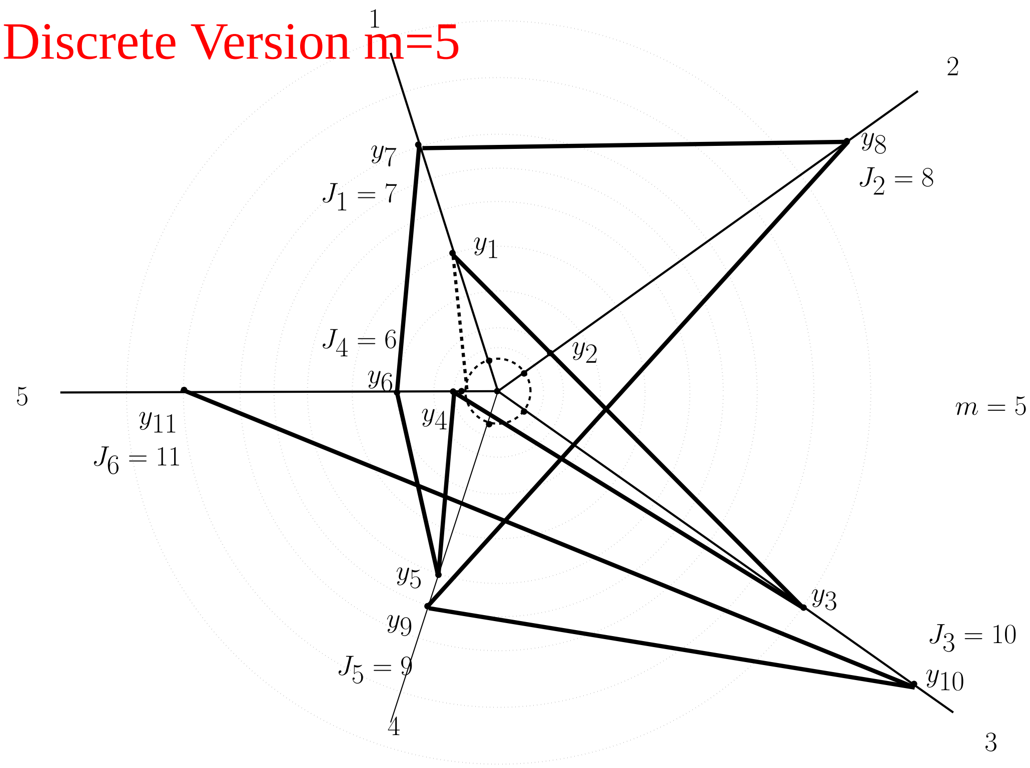


II. 2-ray Version

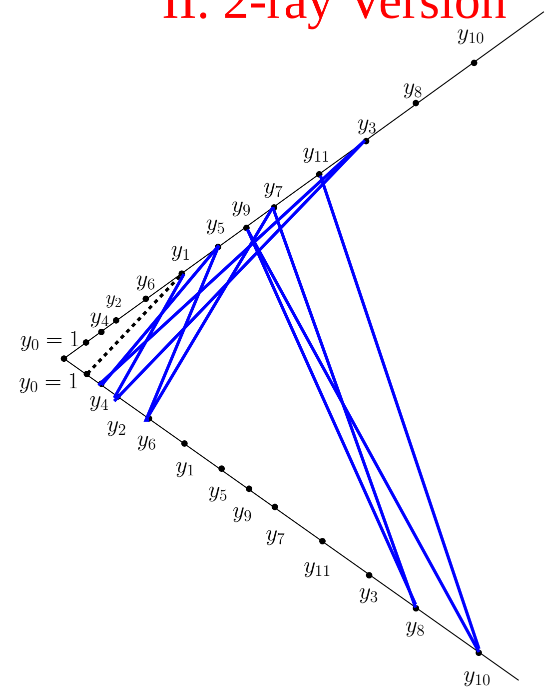


Lower bound construction: General overview

I. Discrete Version m=5

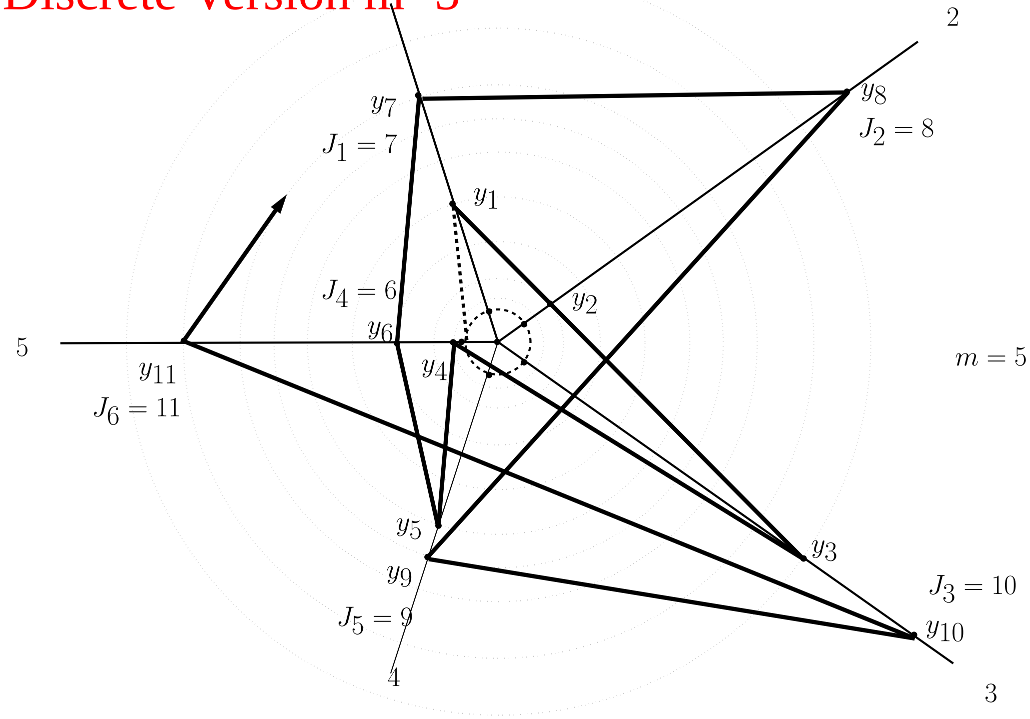


II. 2-ray Version

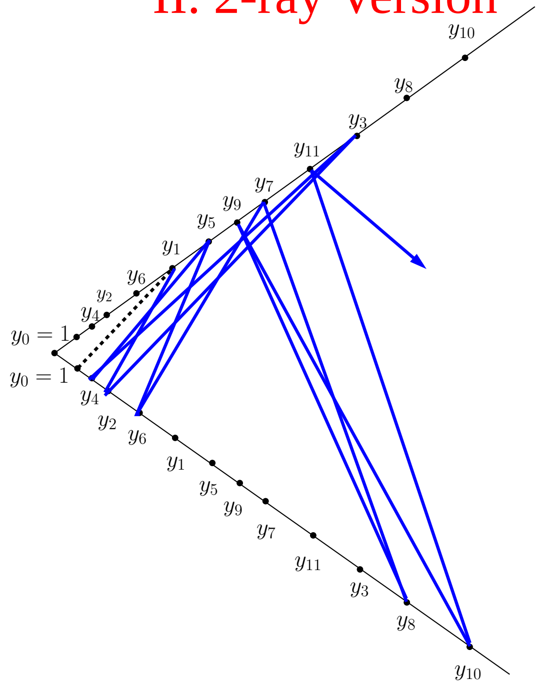


Lower bound construction: General overview

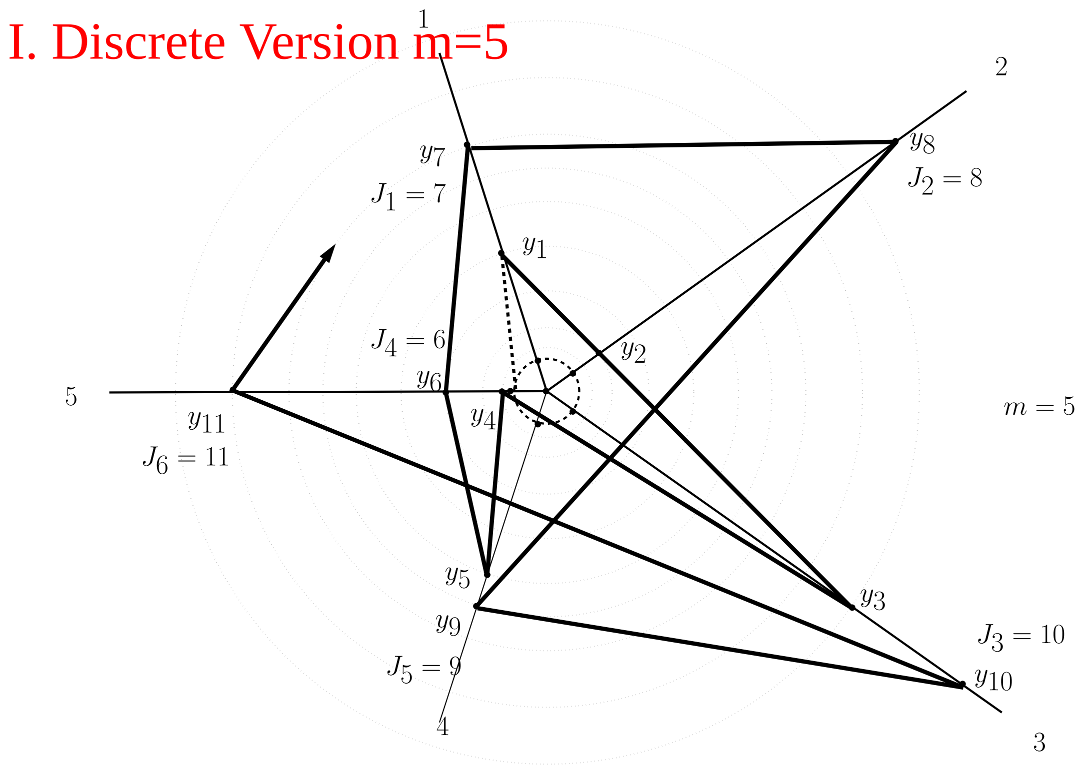
I. Discrete Version m=5

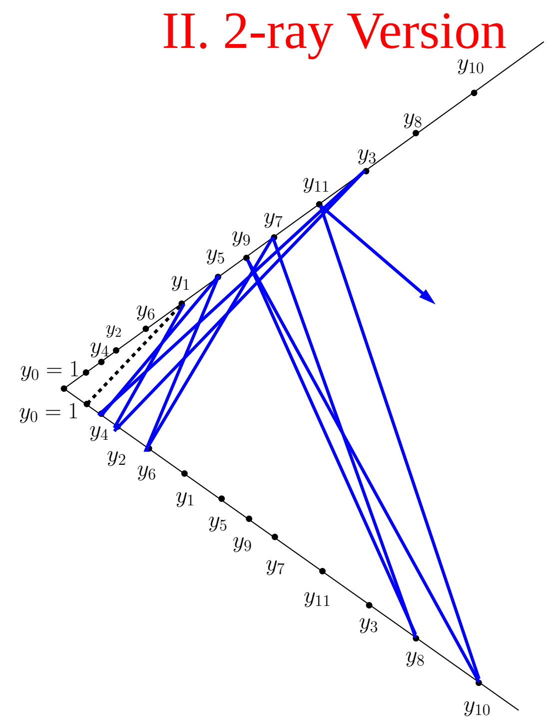


II. 2-ray Version



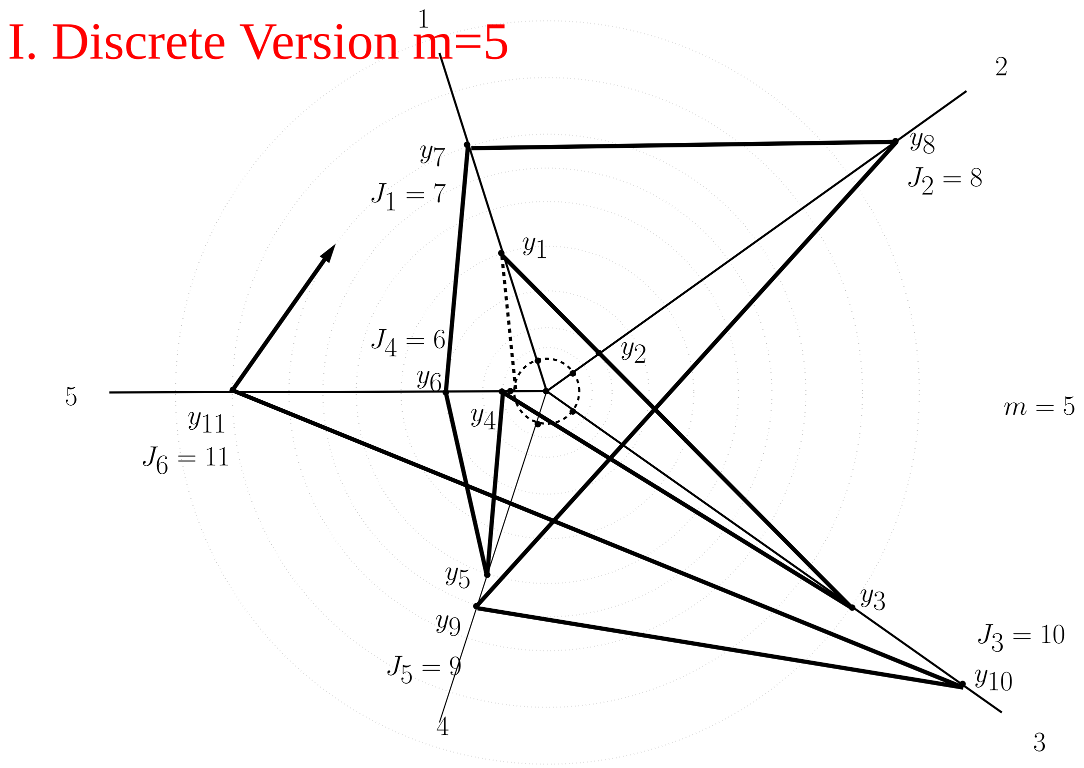
Lower bound construction: General overview

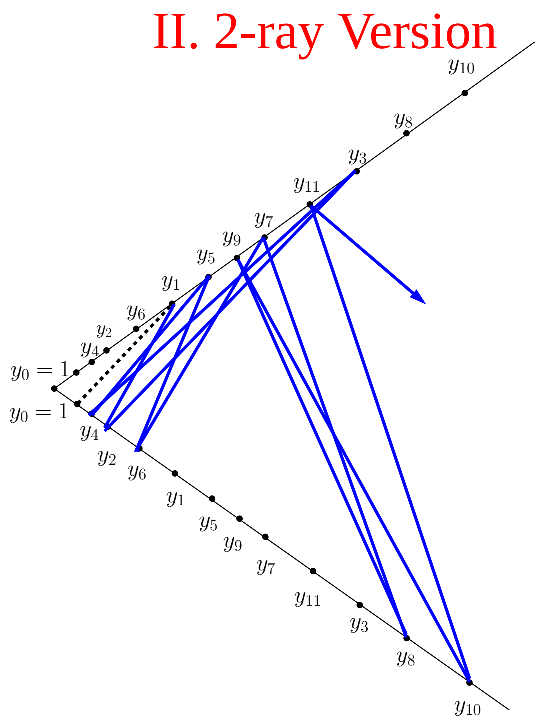




The optimal solution of the 2-ray version is a lower bound on the m-ray version *and* the original version.

Lower bound construction: General overview



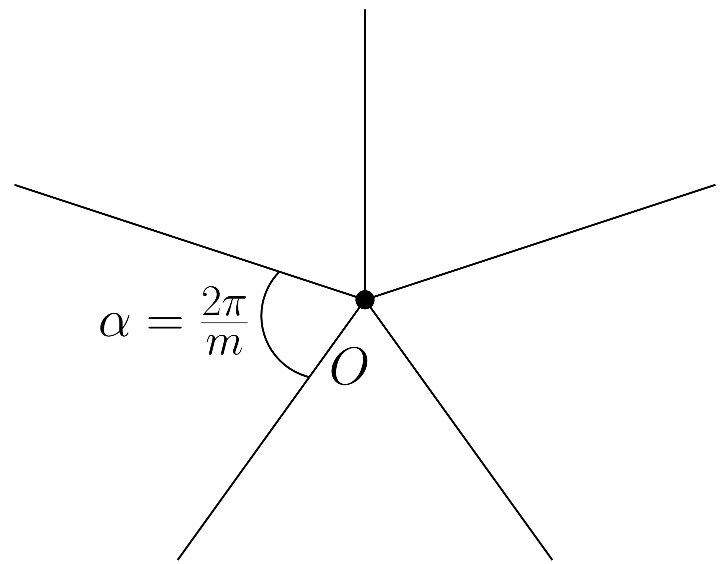


The optimal solution of the 2-ray version is a lower bound on the m-ray version *and* the original version. Use the framework!

Lower bound construction: Discrete version

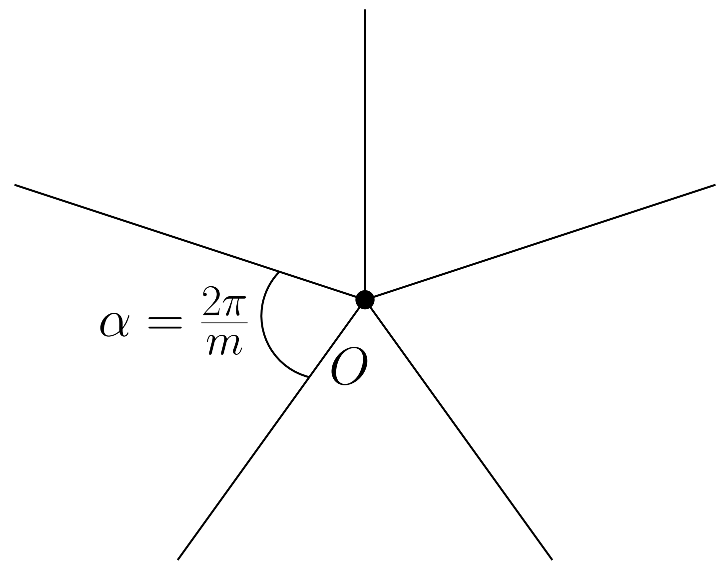
Lower bound construction: Discrete version

- Bundle of m rays



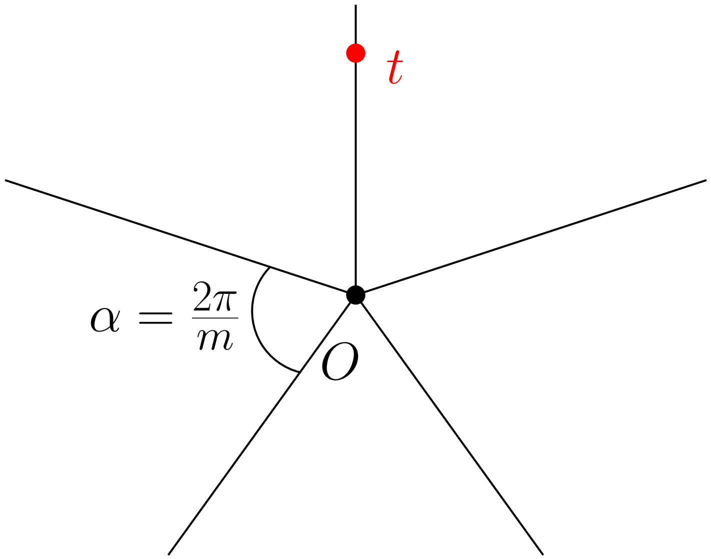
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$



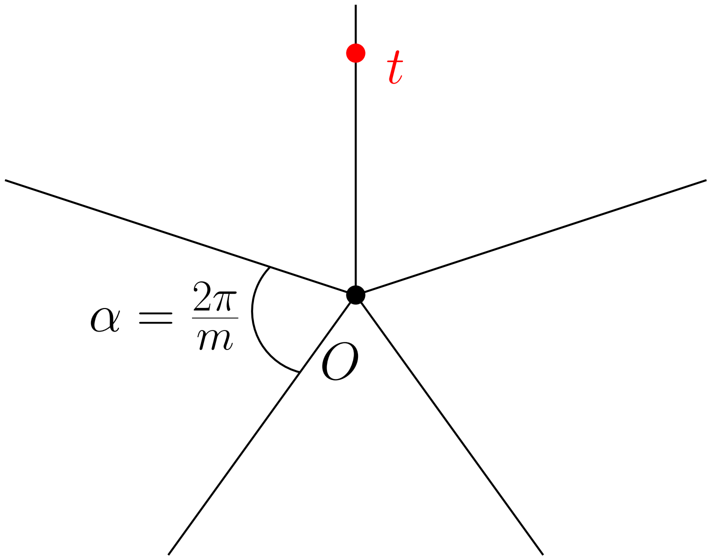
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray



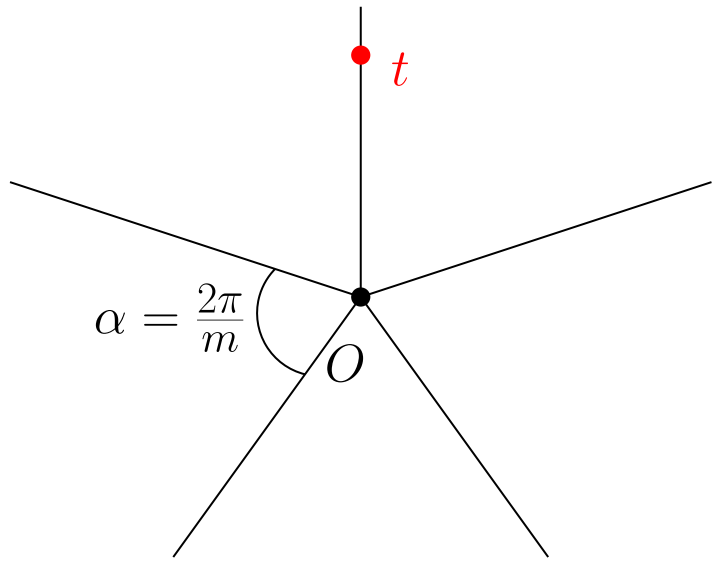
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end



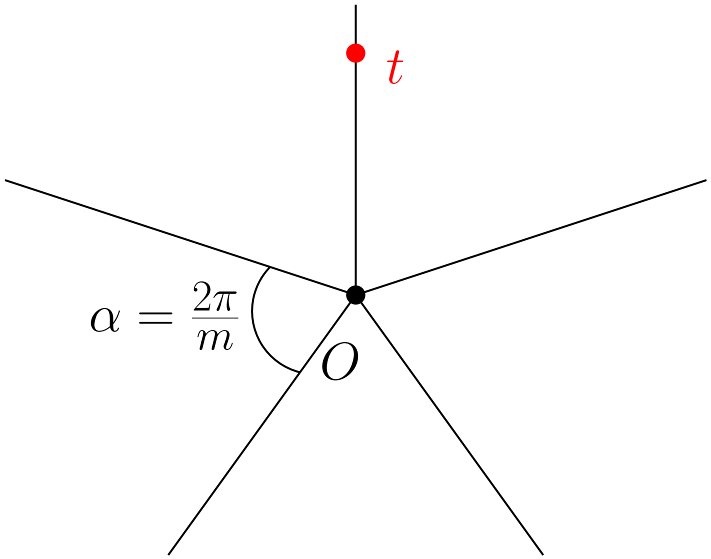
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies



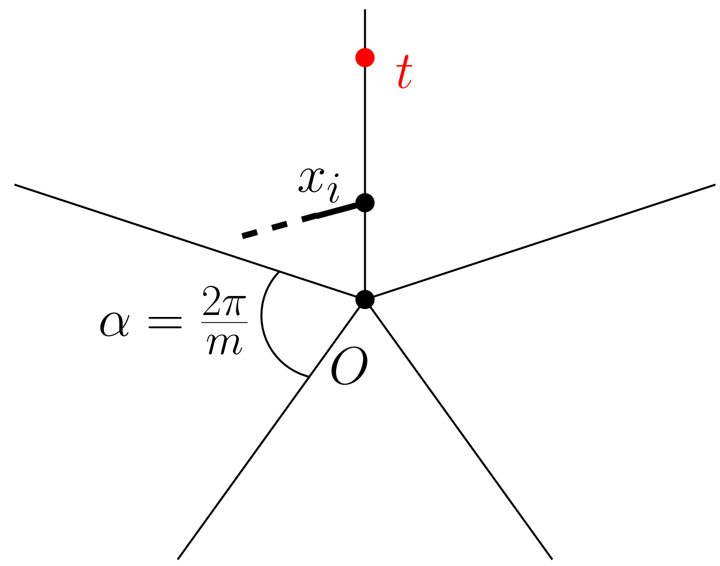
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order



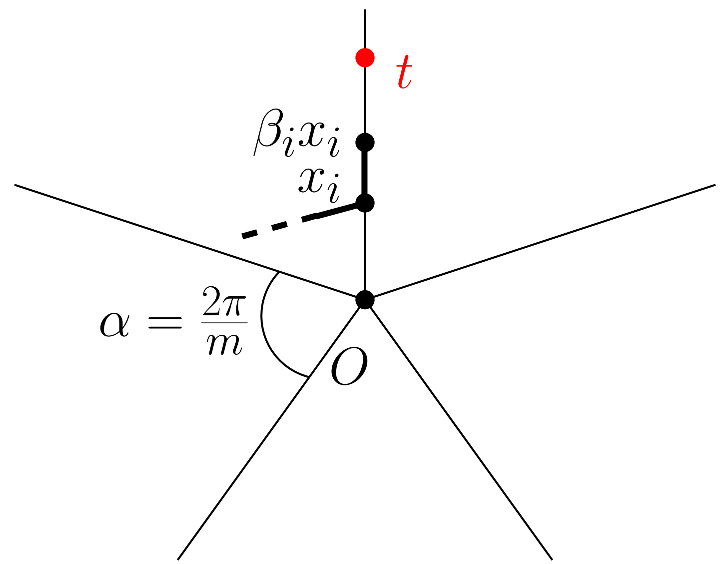
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)
- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$



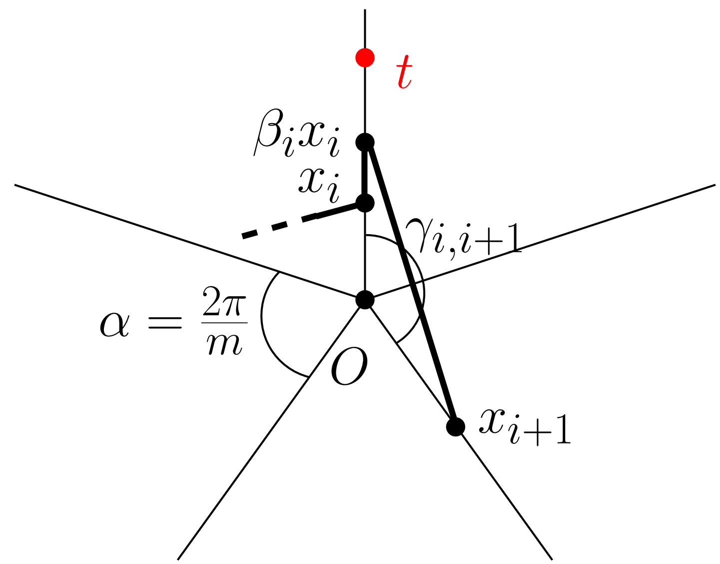
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)
- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$



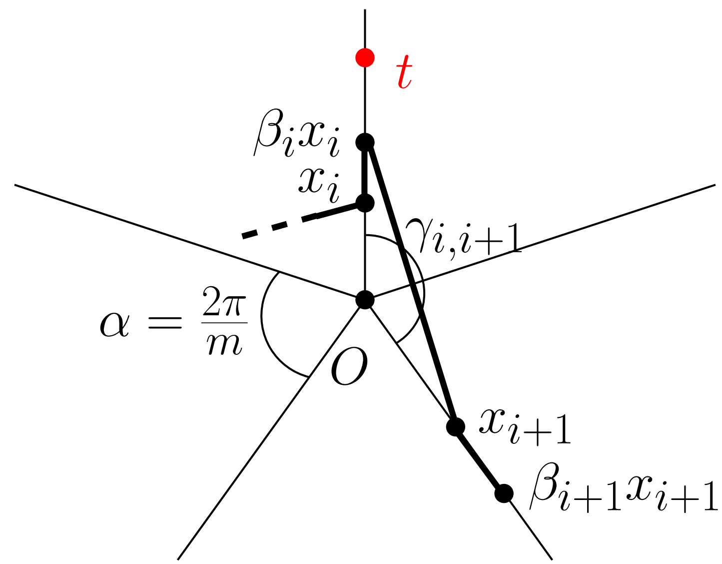
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)
- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$



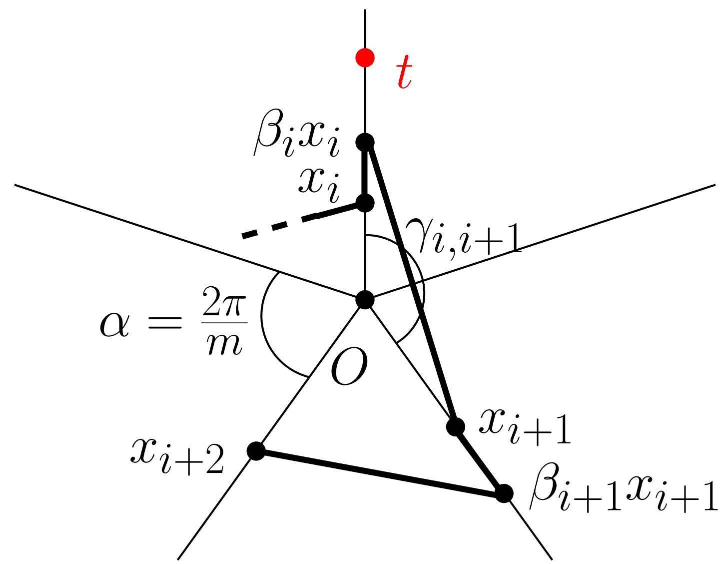
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)
- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$



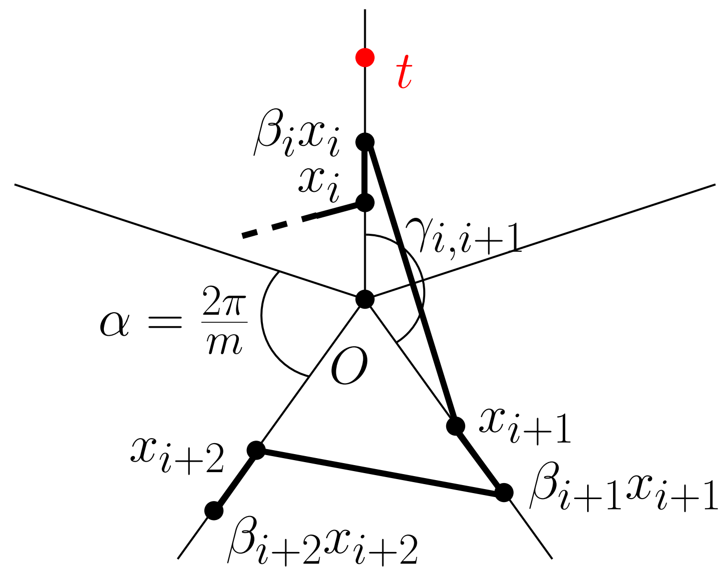
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)
- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$



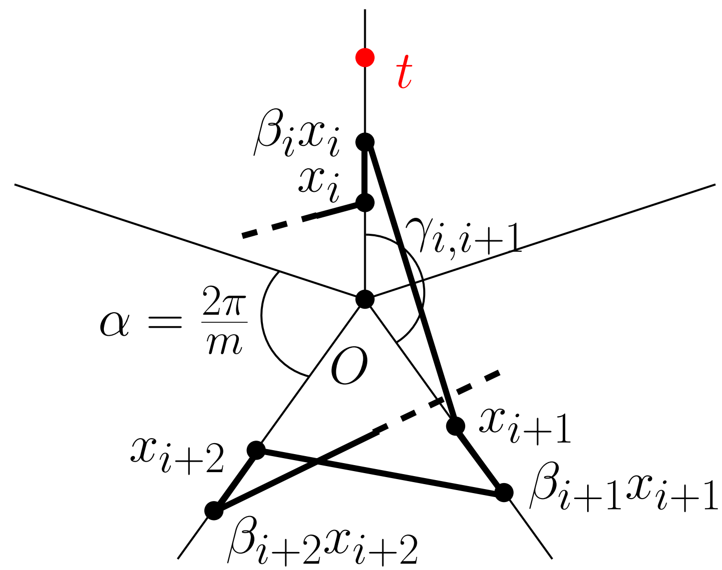
Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)
- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$

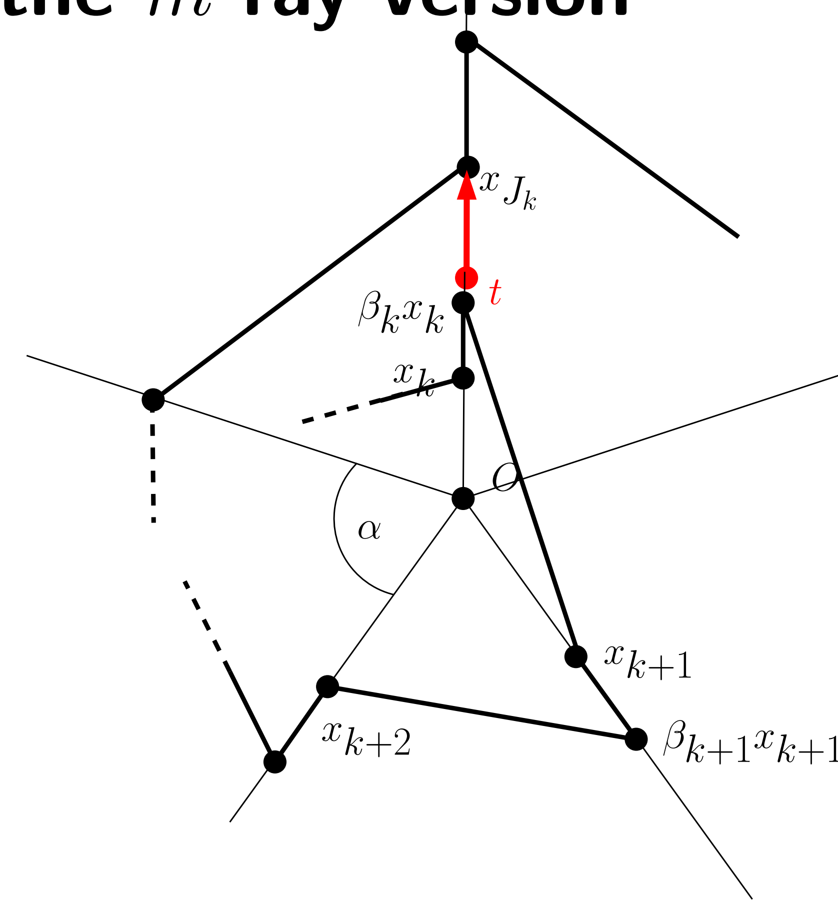


Lower bound construction: Discrete version

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)
- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$

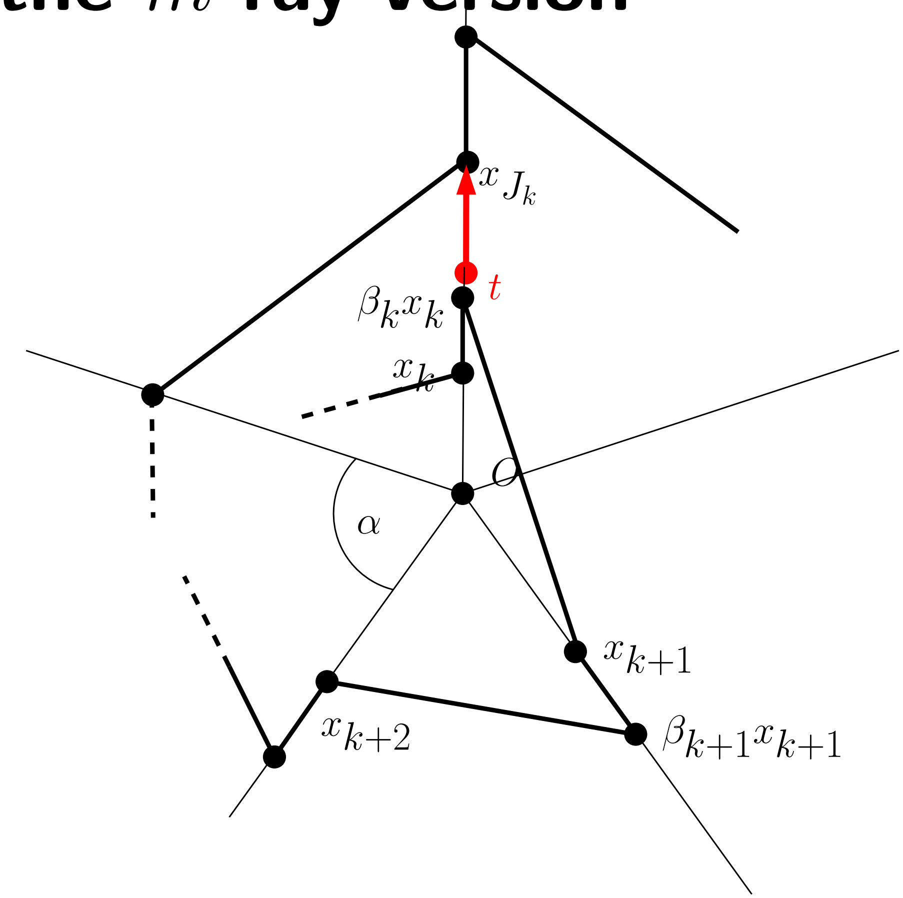


Functional for the m-ray version



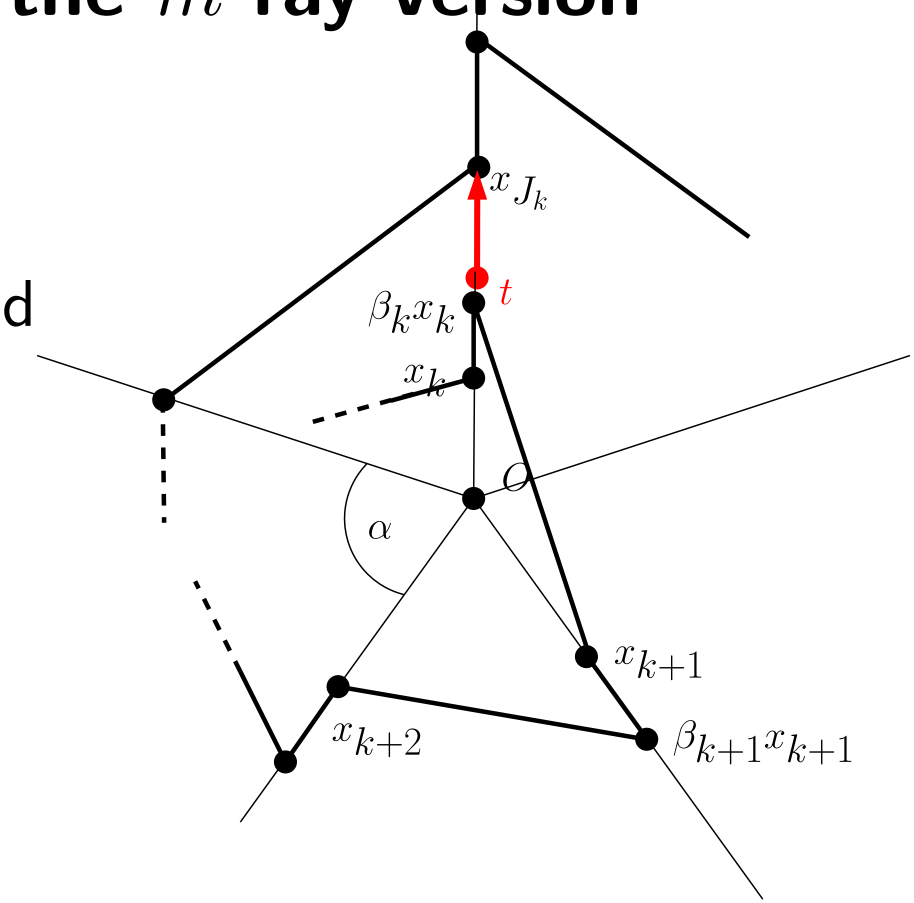
Functional for the m-ray version

- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$



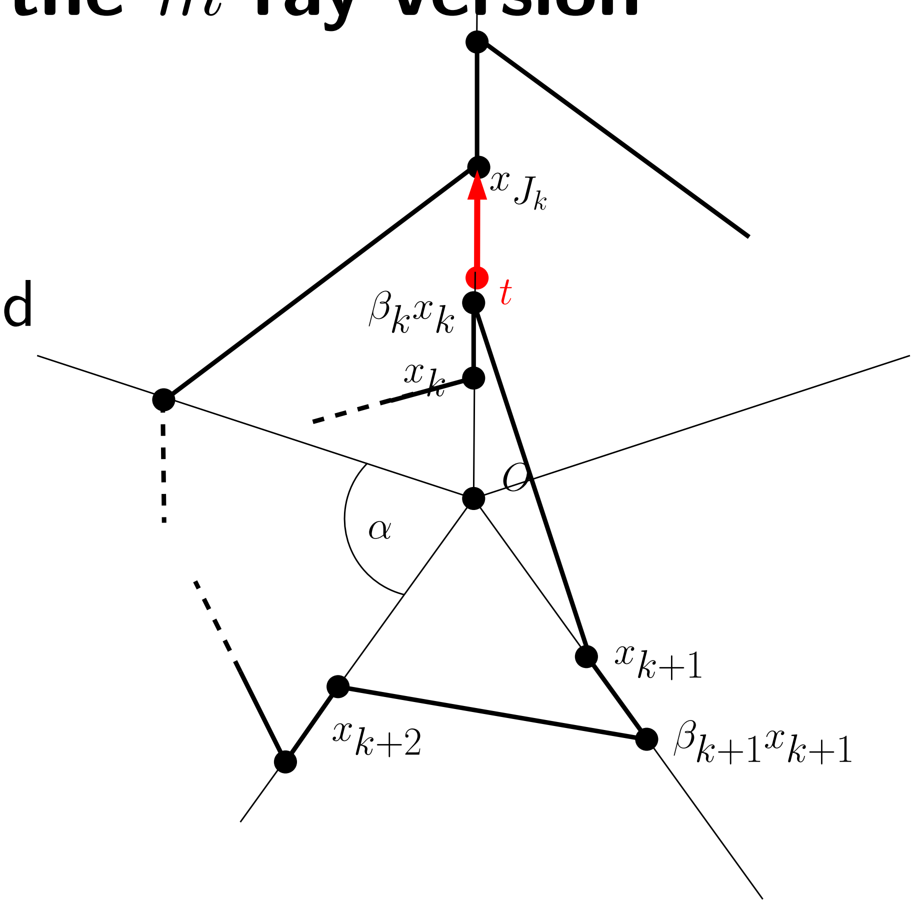
Functional for the m-ray version

- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$
- Find t on a ray at step J_k visited at step k at the last time



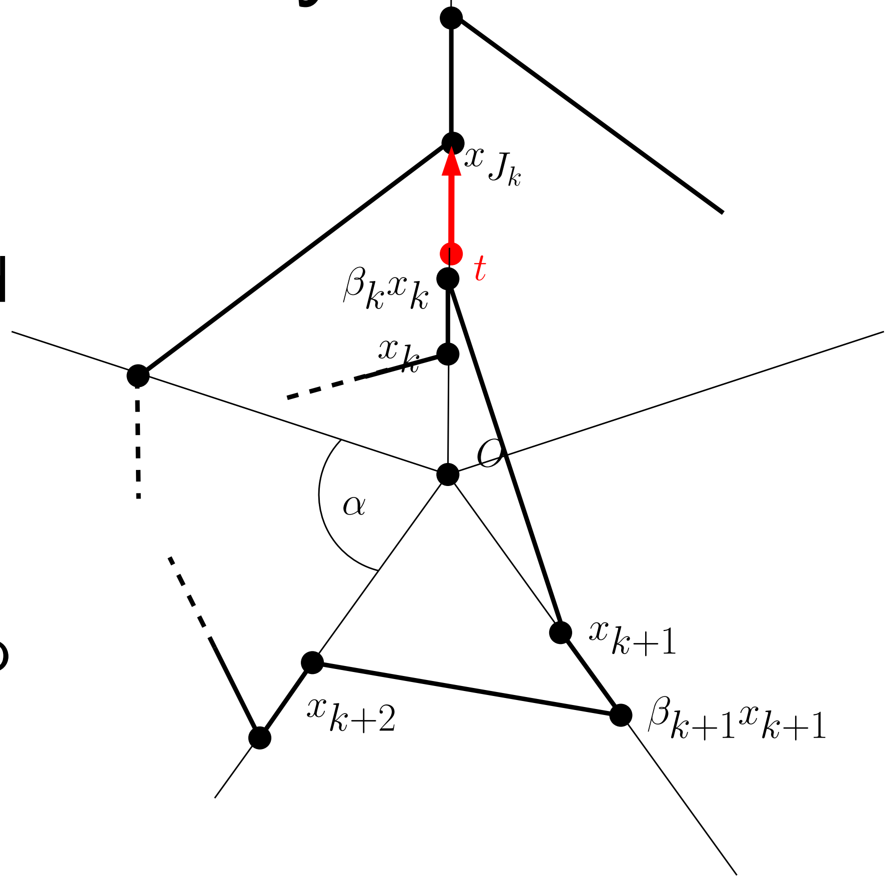
Functional for the m-ray version

- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$
- Find t on a ray at step J_k visited at step k at the last time
- Any order is possible: $i = J_k$, k?

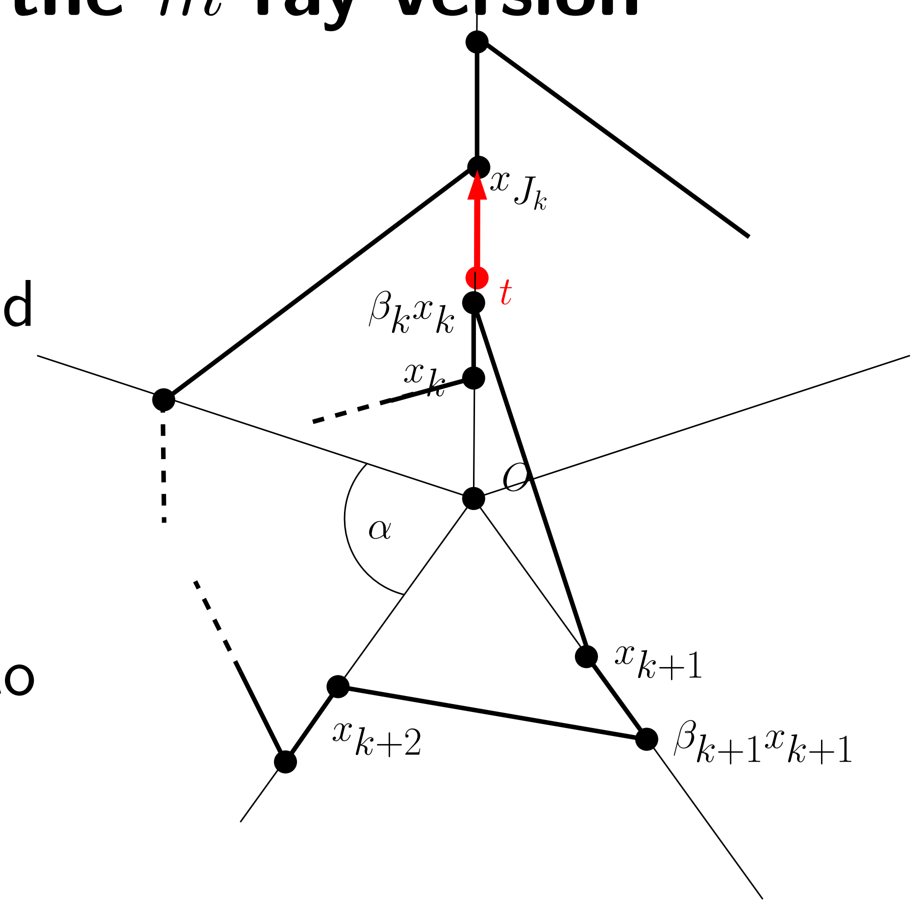


Functional for the m-ray version

- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$
- Find t on a ray at step J_k visited at step k at the last time
- Any order is possible: $i = J_k$, k?
- Worst-case for J_k: t close to $\beta_k x_k$



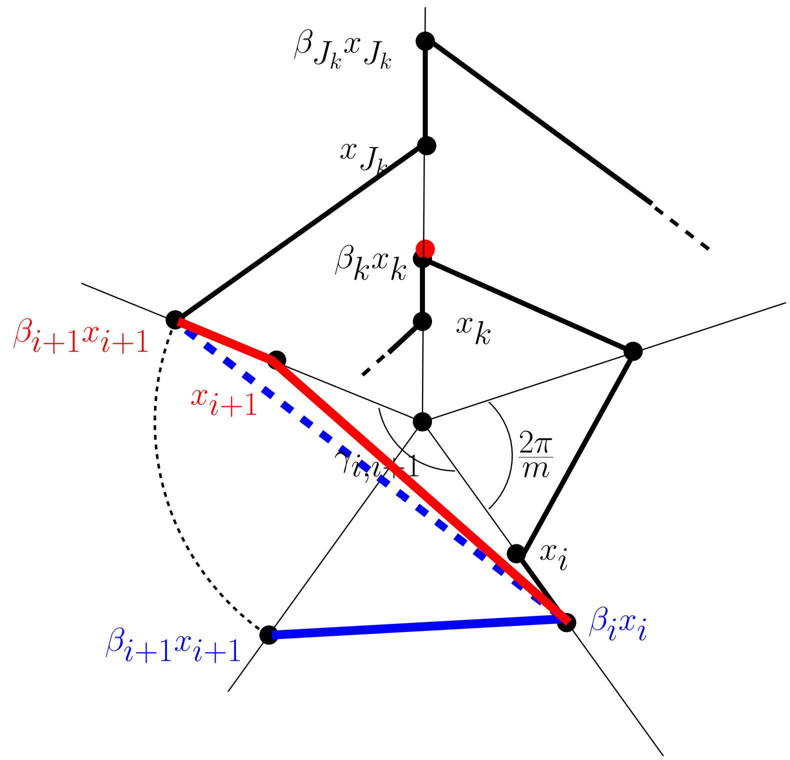
Functional for the m-ray version



- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$
- Find t on a ray at step J_k visited at step k at the last time
- Any order is possible: $i = J_k,\ k$?
- Worst-case for J_k: t close to $\beta_k x_k$
- Ratio: $C(S) = \sup_k$

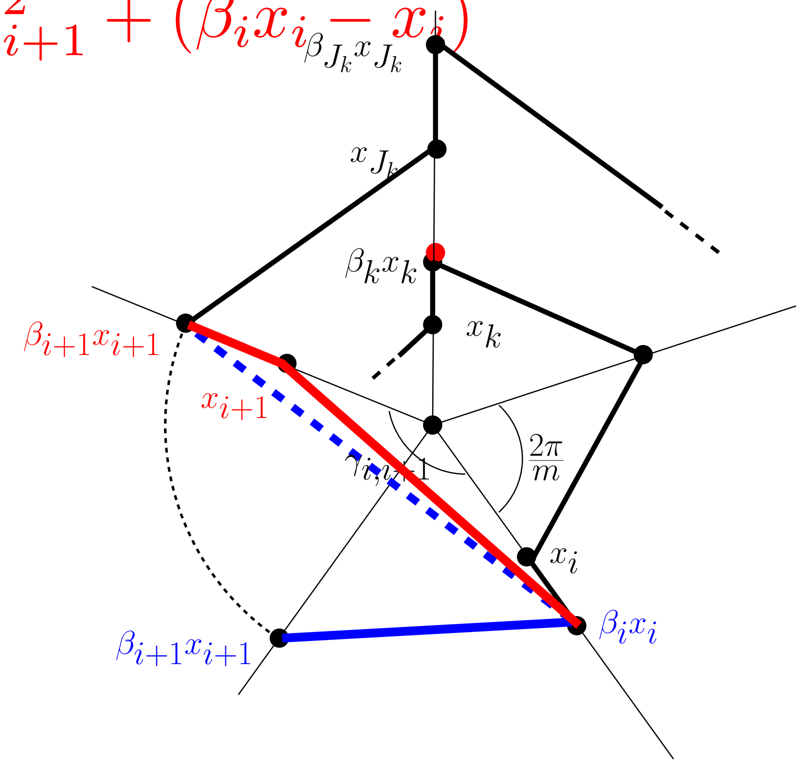
$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)}{\beta_k x_k}$$

From m rays toward 2 rays



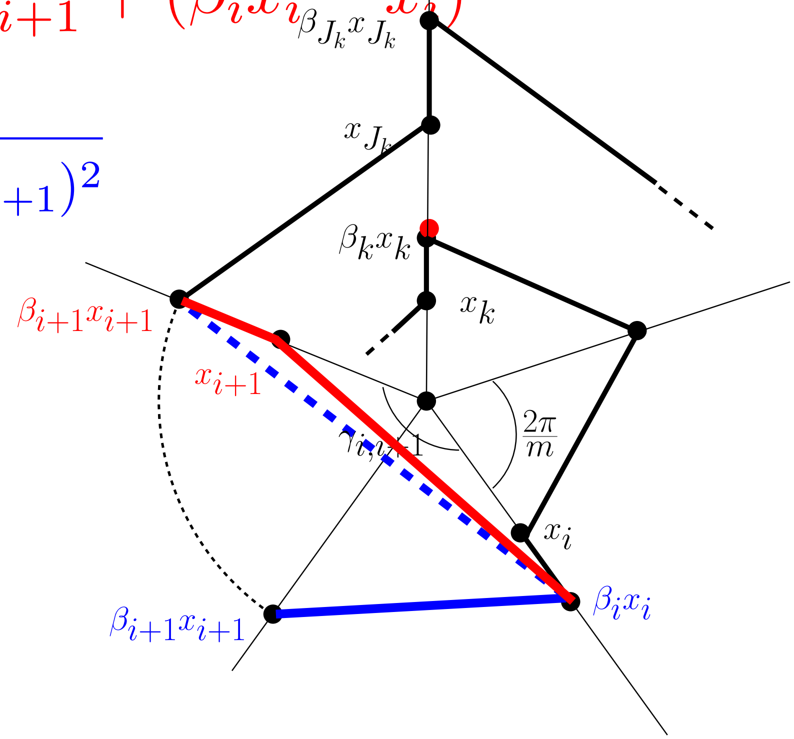
From m rays toward 2 rays

- $\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos\gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)$



From m rays toward 2 rays

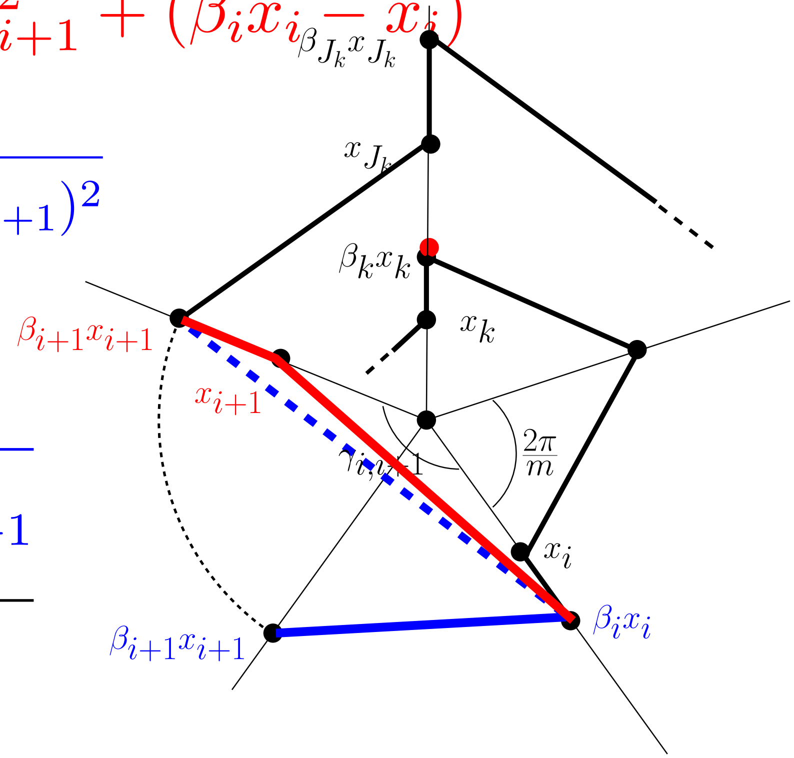
- $\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)$
- Shrinks to:
  $\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i \beta_{i+1} x_{i+1} \cos \frac{2\pi}{m} + (\beta_{i+1} x_{i+1})^2}$



From m rays toward 2 rays

- $\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)$
- Shrinks to:
$\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i \beta_{i+1} x_{i+1} \cos \frac{2\pi}{m} + (\beta_{i+1} x_{i+1})^2}$
- Lower bound: $C(S) \geq$

$$\sup_k \frac{\sum_{i=1}^{J_k - 2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$

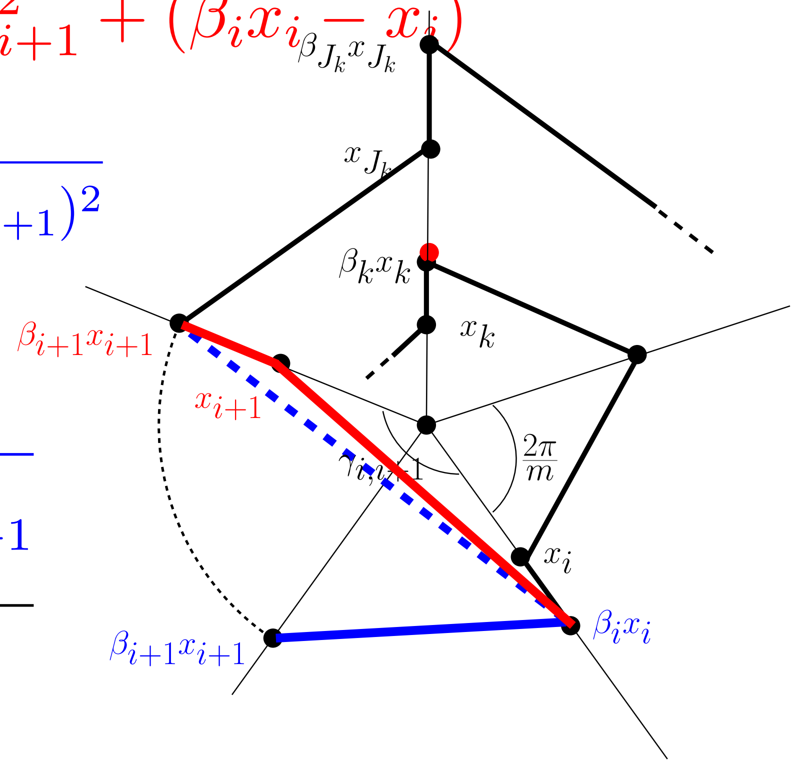


From m rays toward 2 rays

- $\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)$
- Shrinks to: $\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i \beta_{i+1} x_{i+1} \cos \frac{2\pi}{m} + (\beta_{i+1} x_{i+1})^2}$
- Lower bound: $C(S) \geq$

$$\sup_k \frac{\sum_{i=1}^{J_k - 2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$

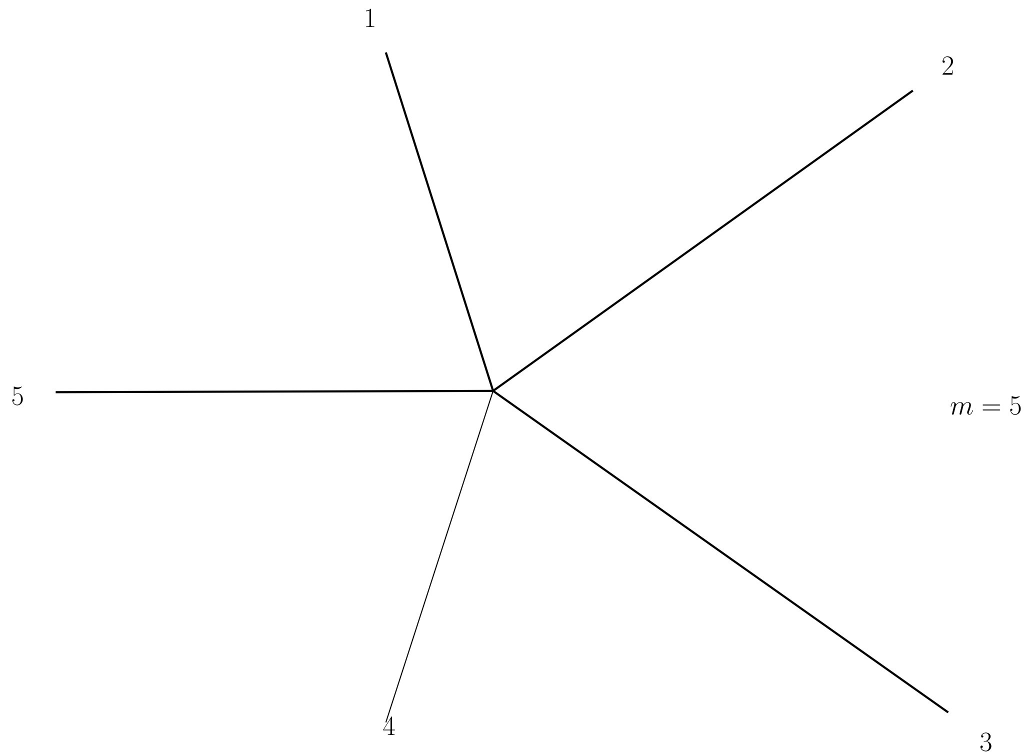
Minimize!!



Example: From m toward 2 rays

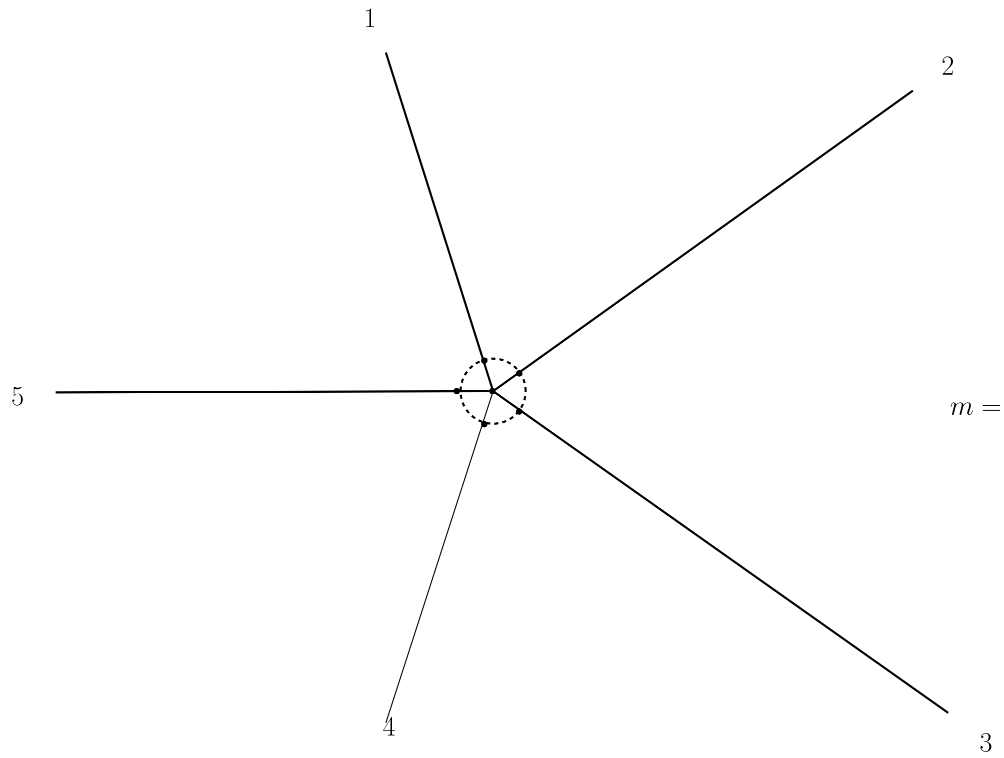
Example: From m toward 2 rays

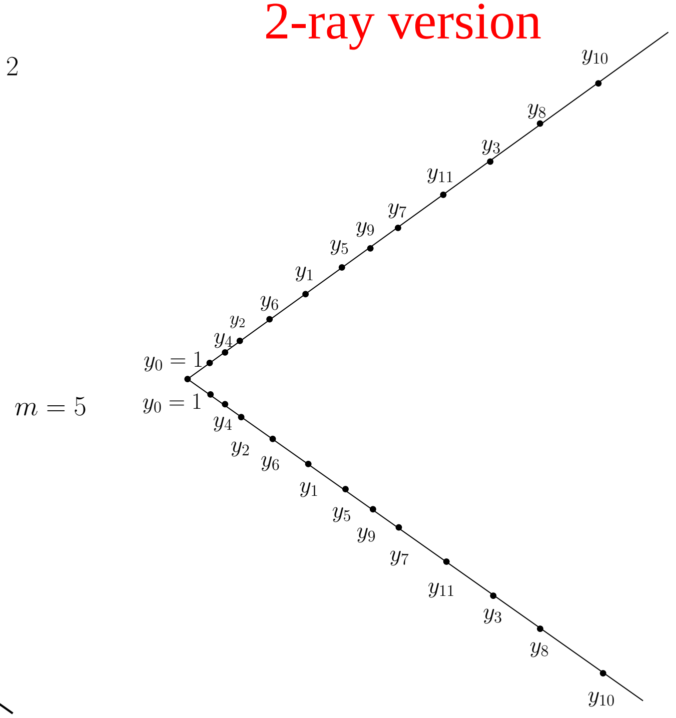
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!



Example: From m toward 2 rays

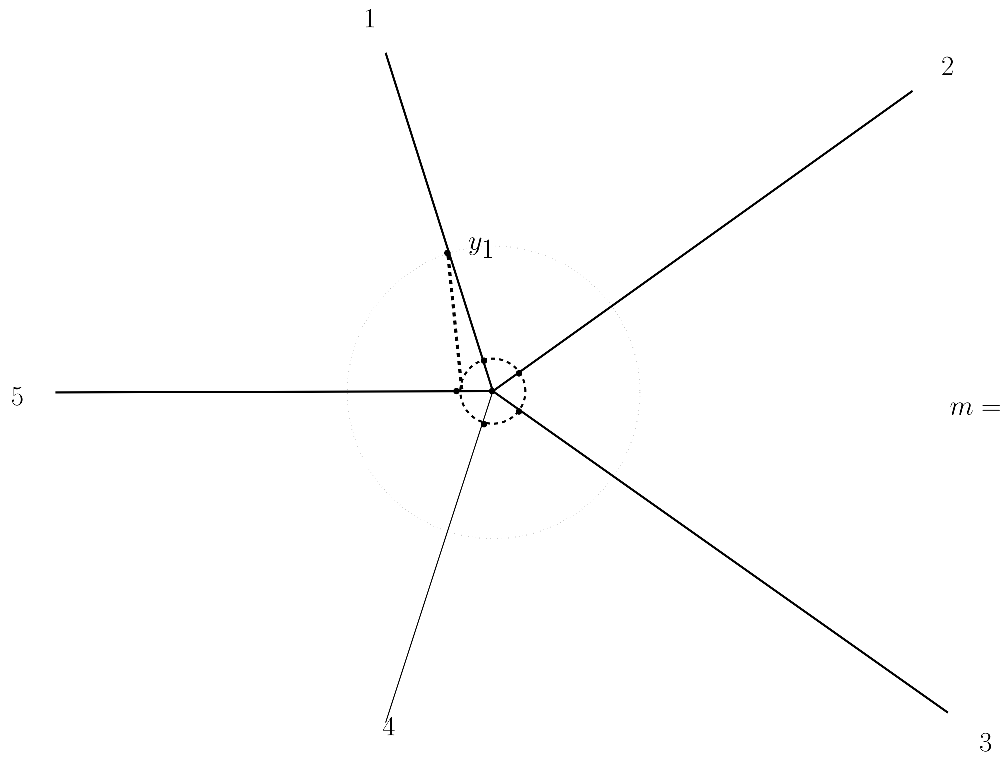
$\sup_k \frac{\sum_{i=1}^{J_k - 2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

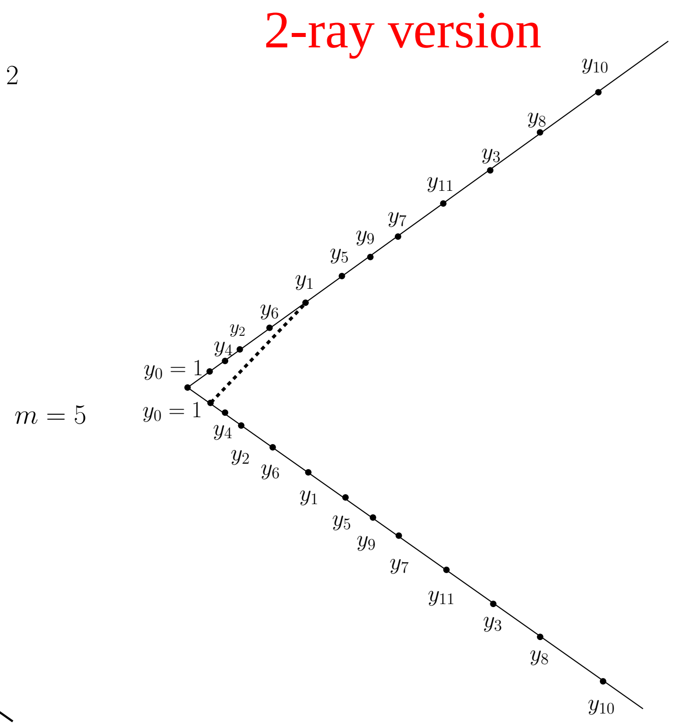




Example: From m toward 2 rays

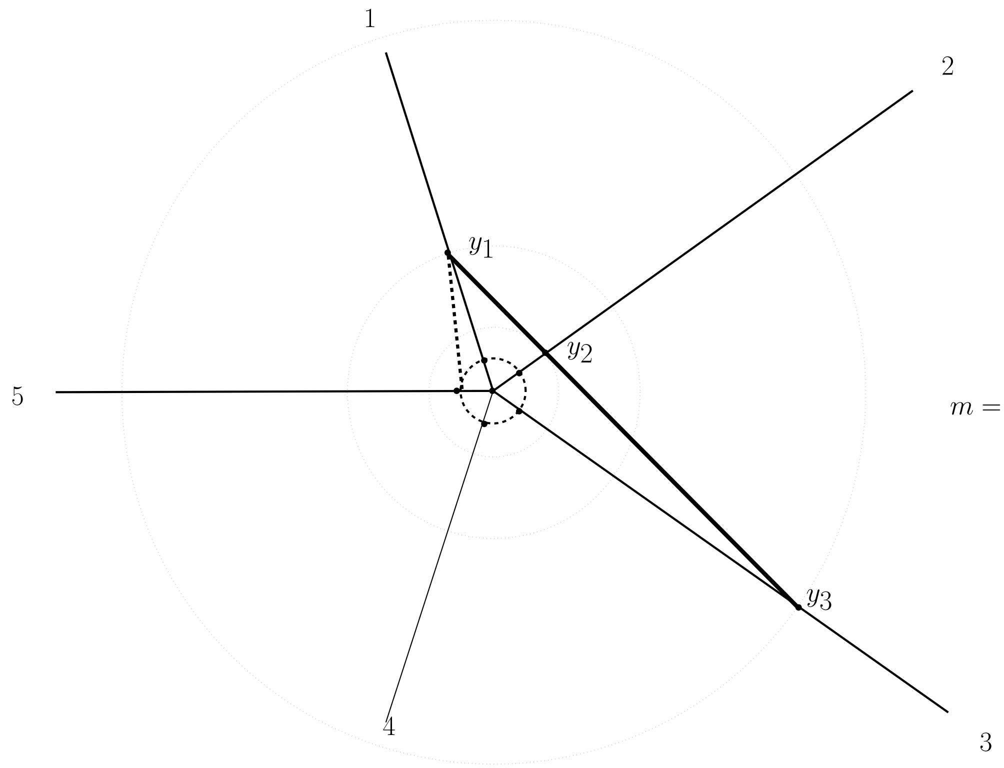
$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, indices J_k original *visiting order*!

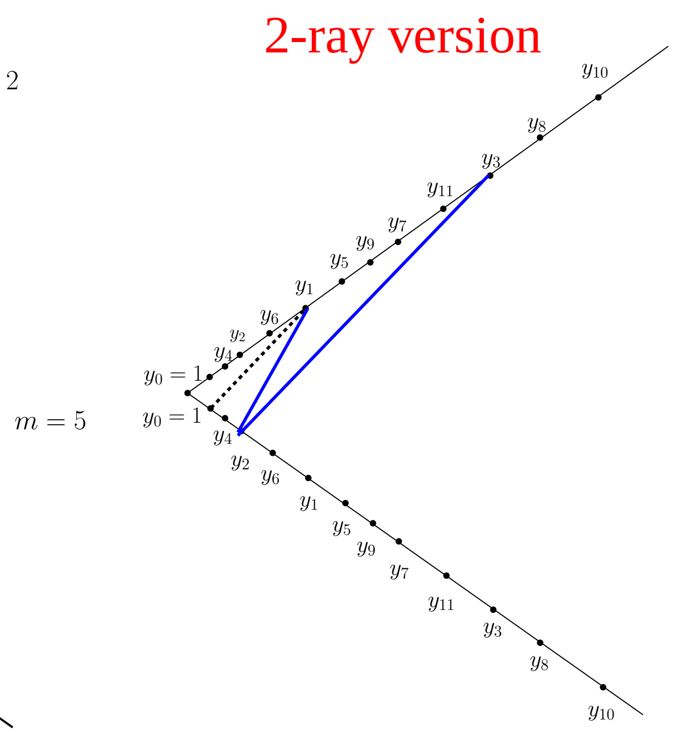




Example: From m toward 2 rays

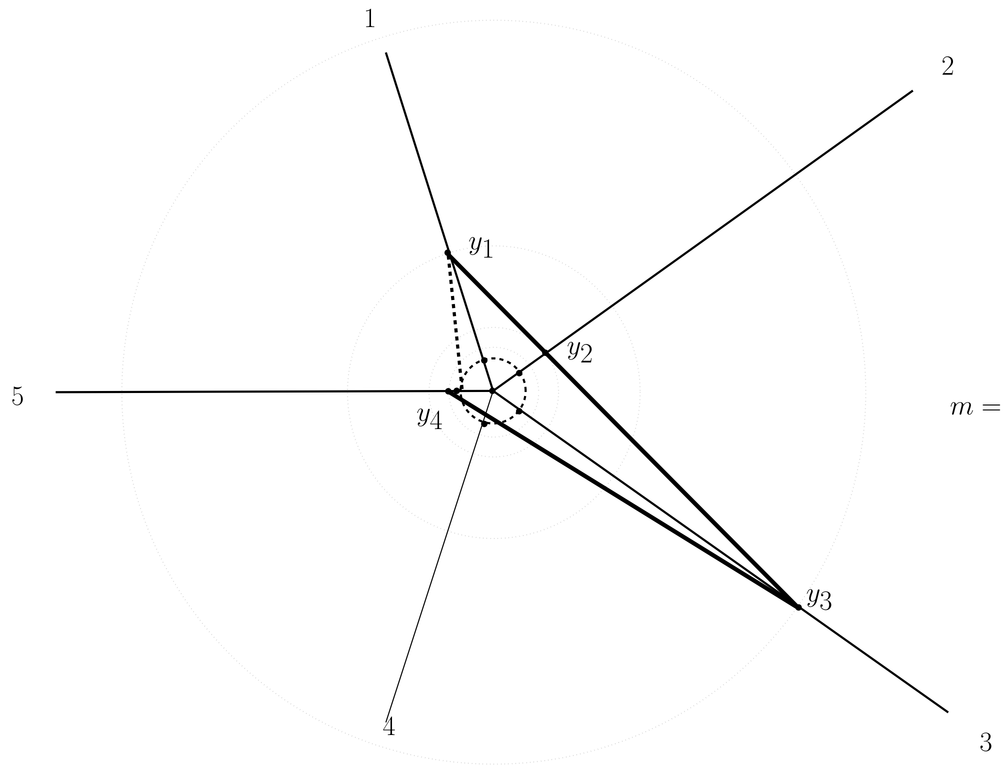
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

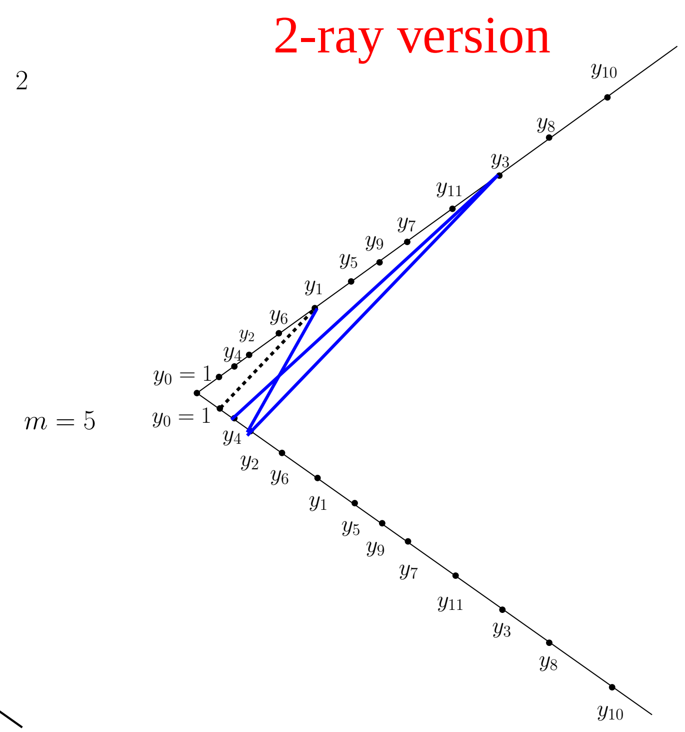




Example: From m toward 2 rays

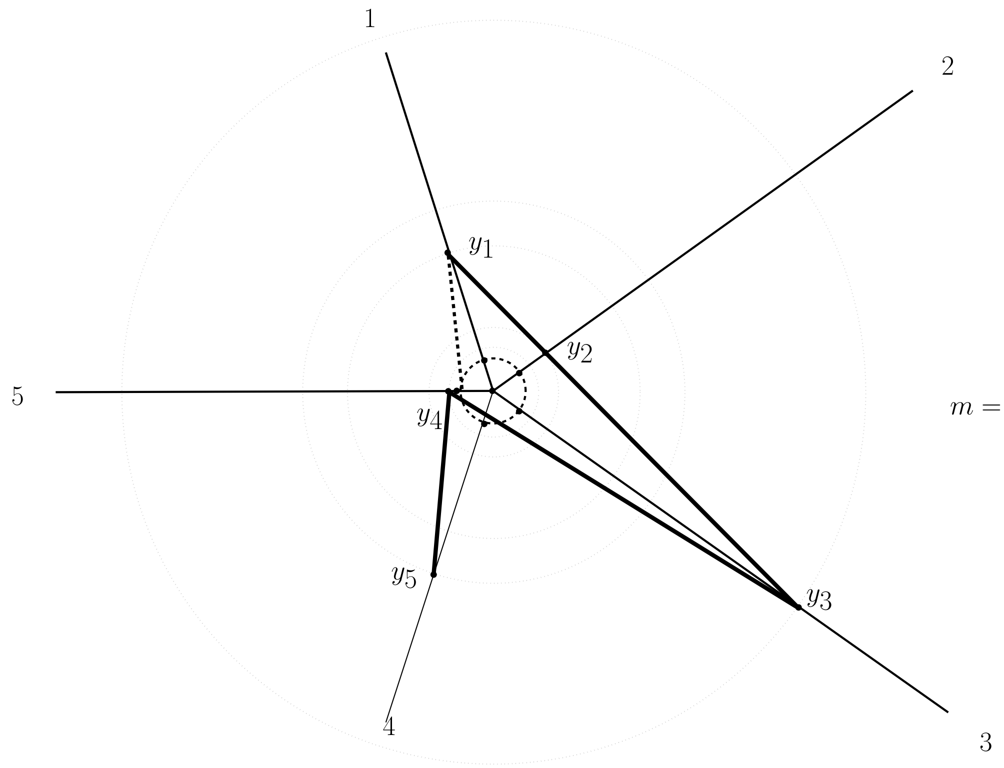
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

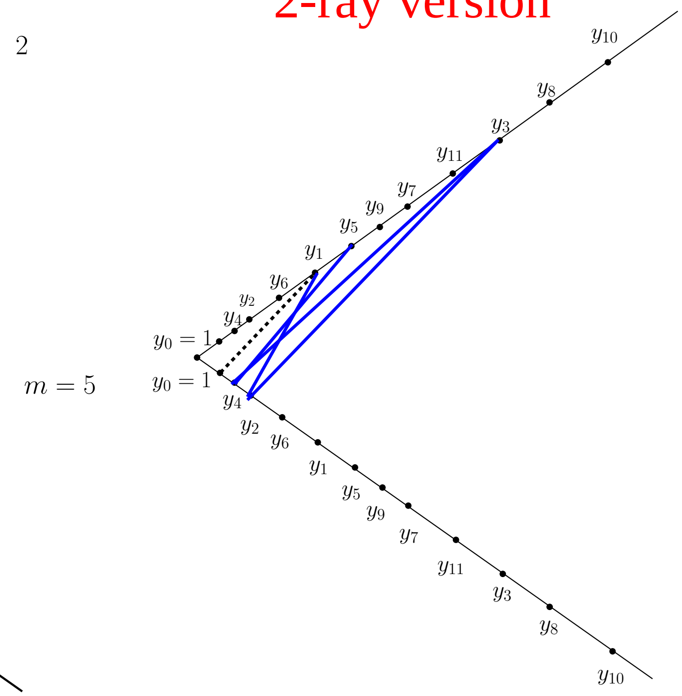




Example: From m toward 2 rays

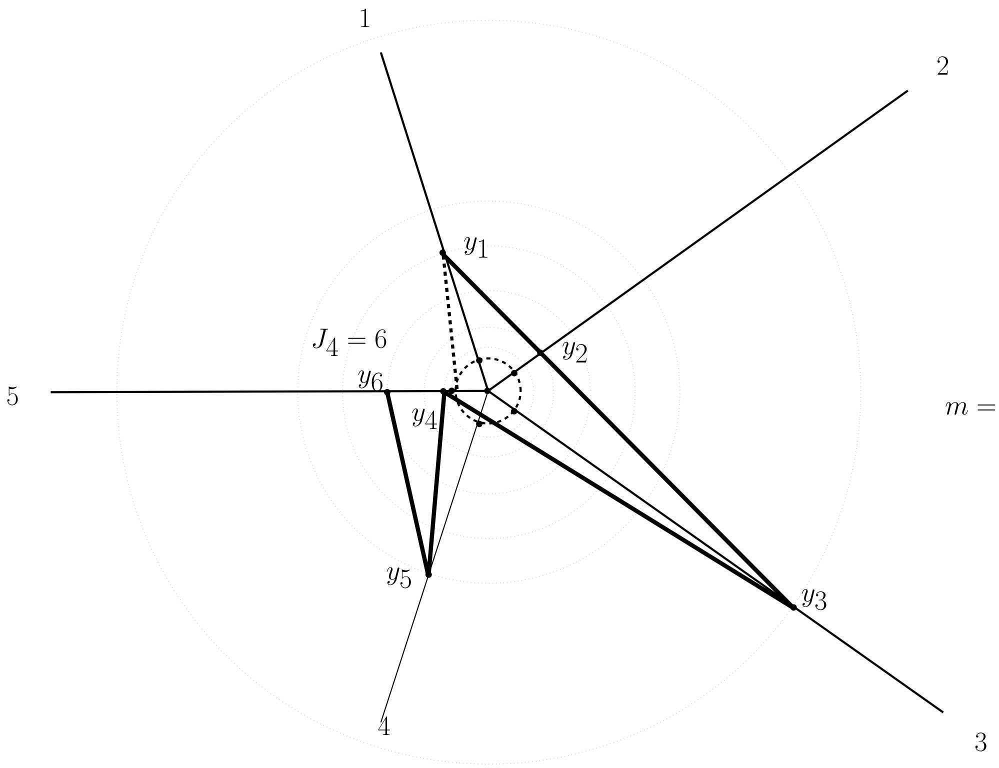
$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, indices J_k original *visiting order*!

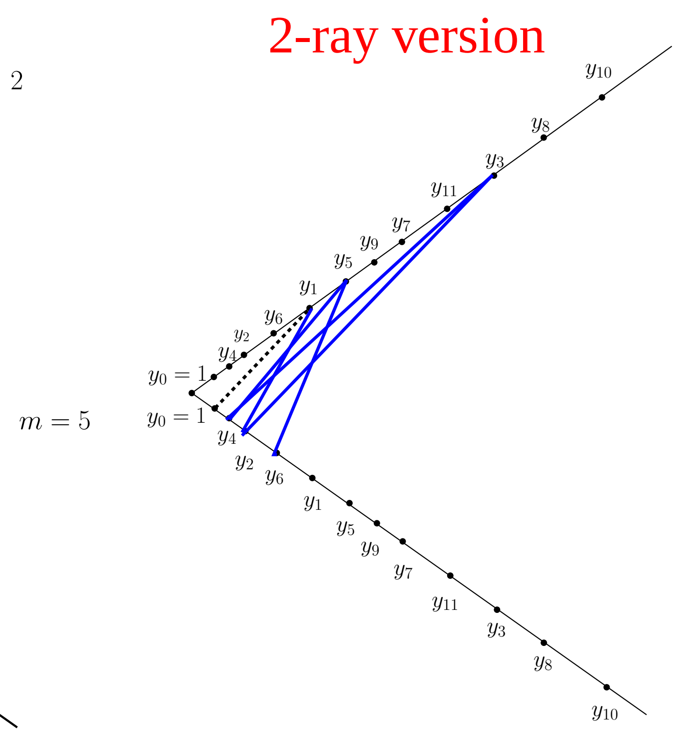




Example: From m toward 2 rays

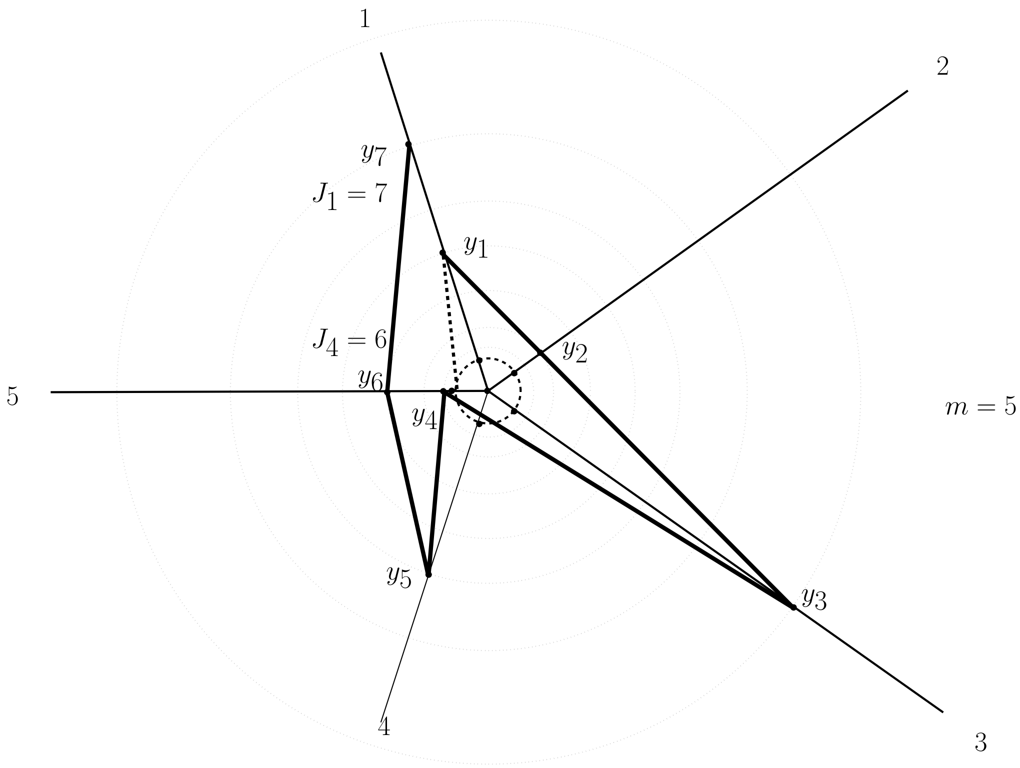
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

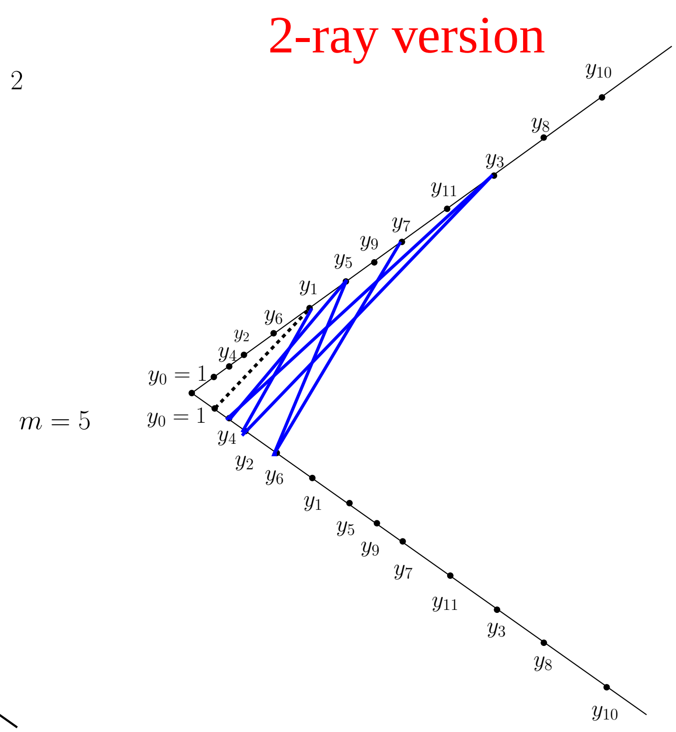




Example: From m toward 2 rays

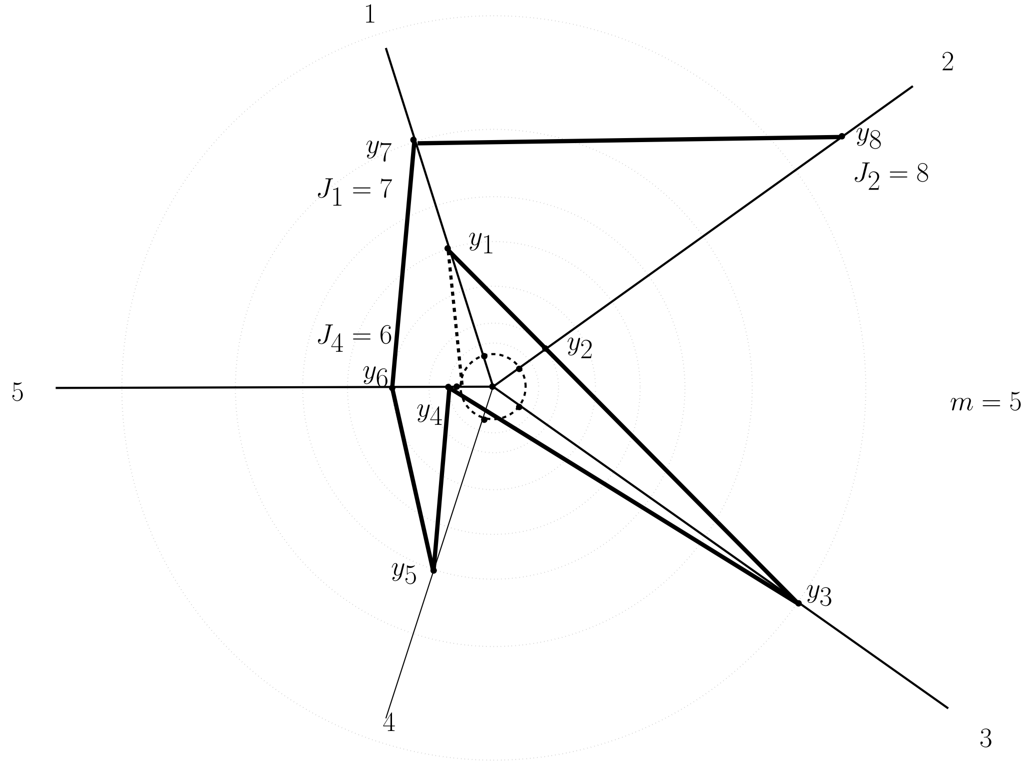
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

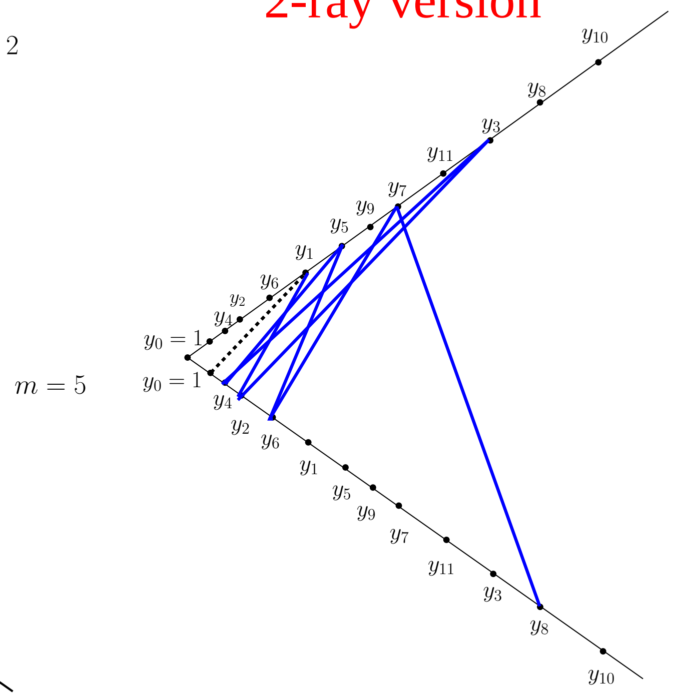




Example: From m toward 2 rays

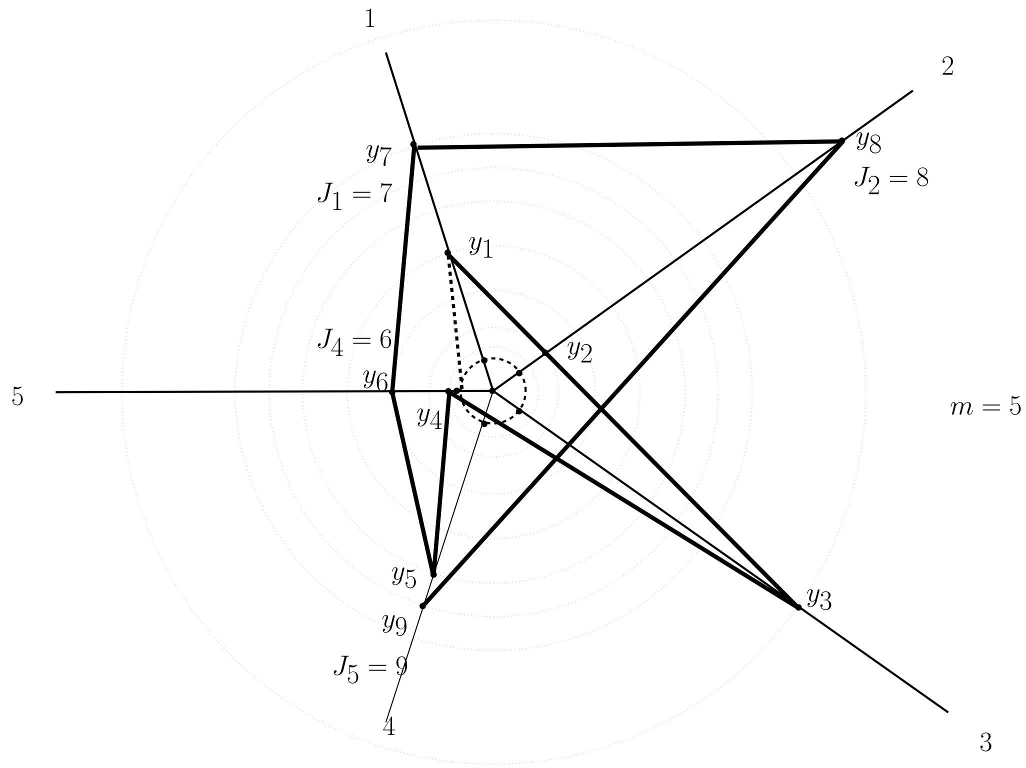
$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1}\cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, indices J_k original *visiting order*!

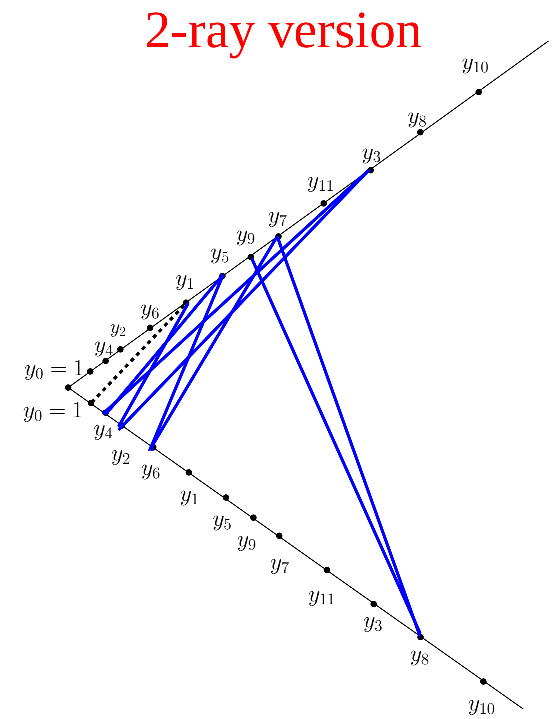




Example: From m toward 2 rays

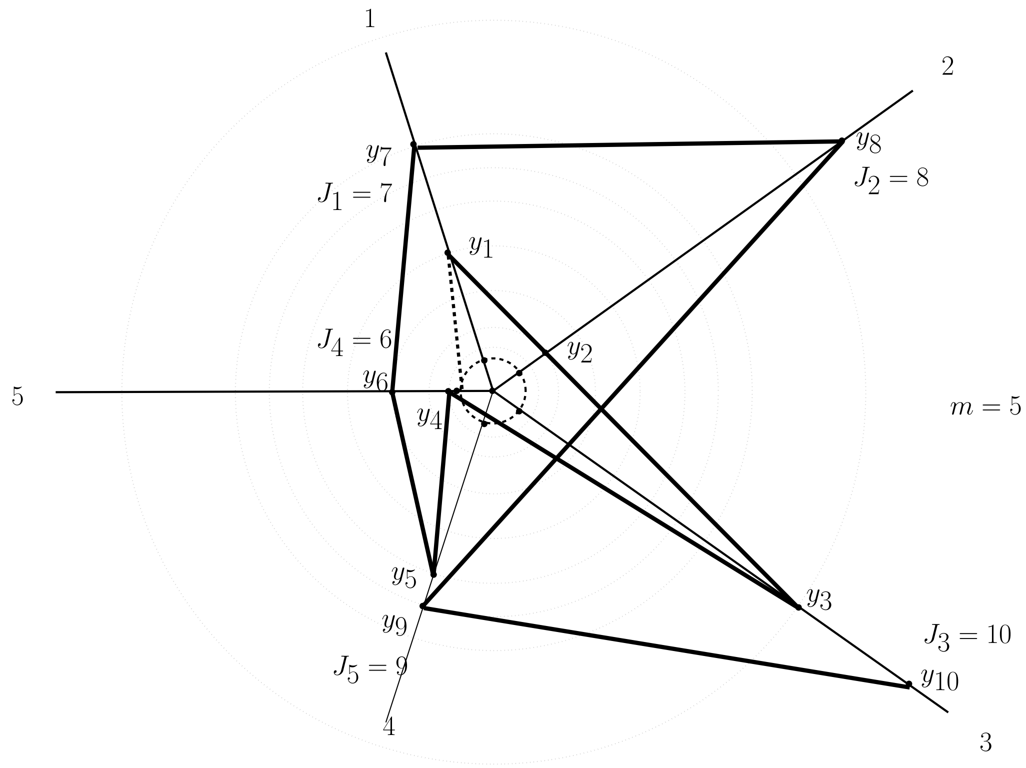
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

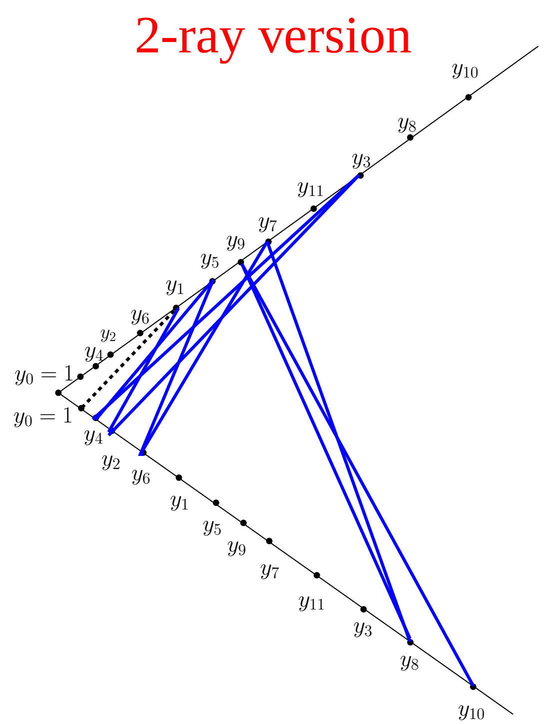




Example: From m toward 2 rays

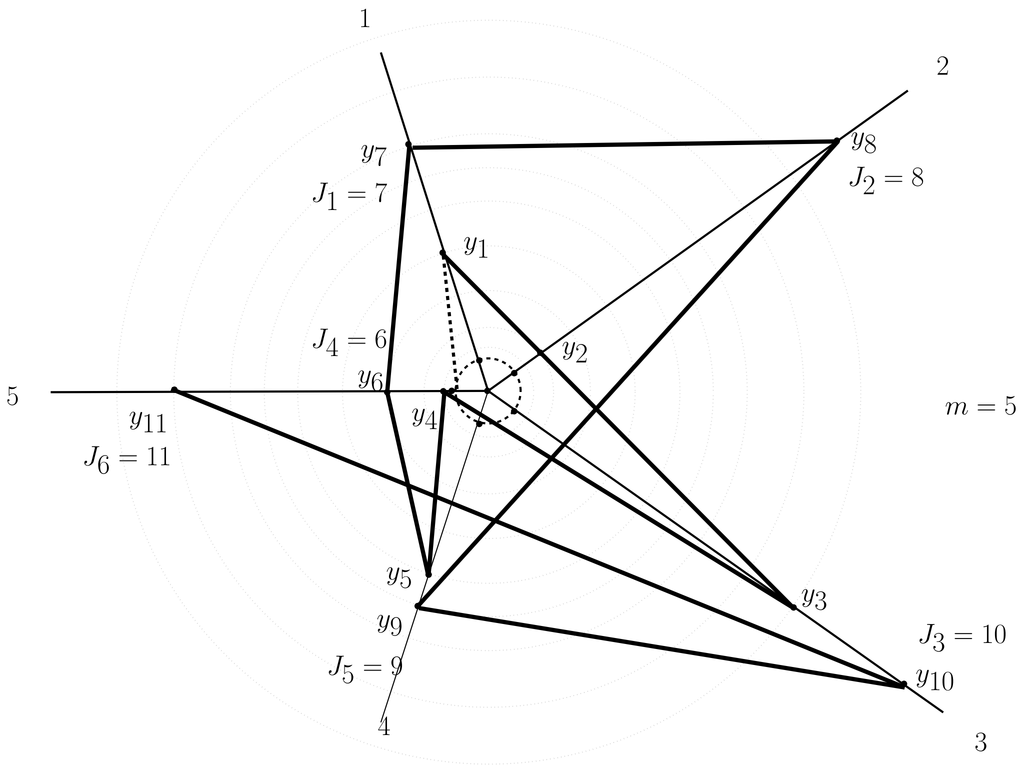
$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, indices J_k original *visiting order*!



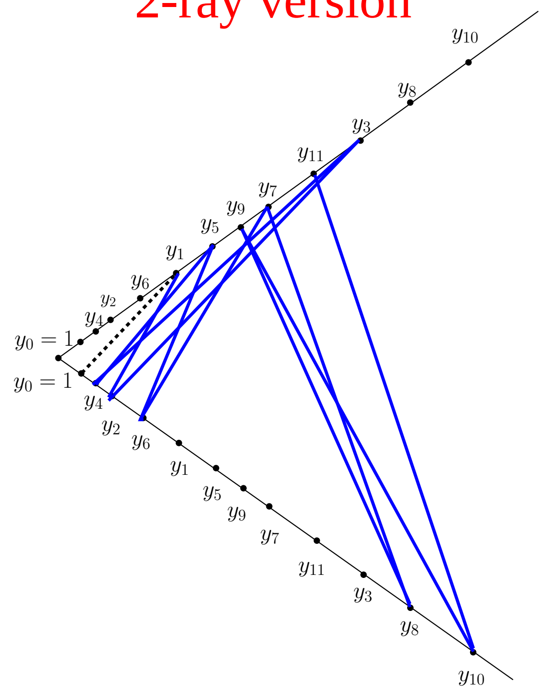


Example: From m toward 2 rays

$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

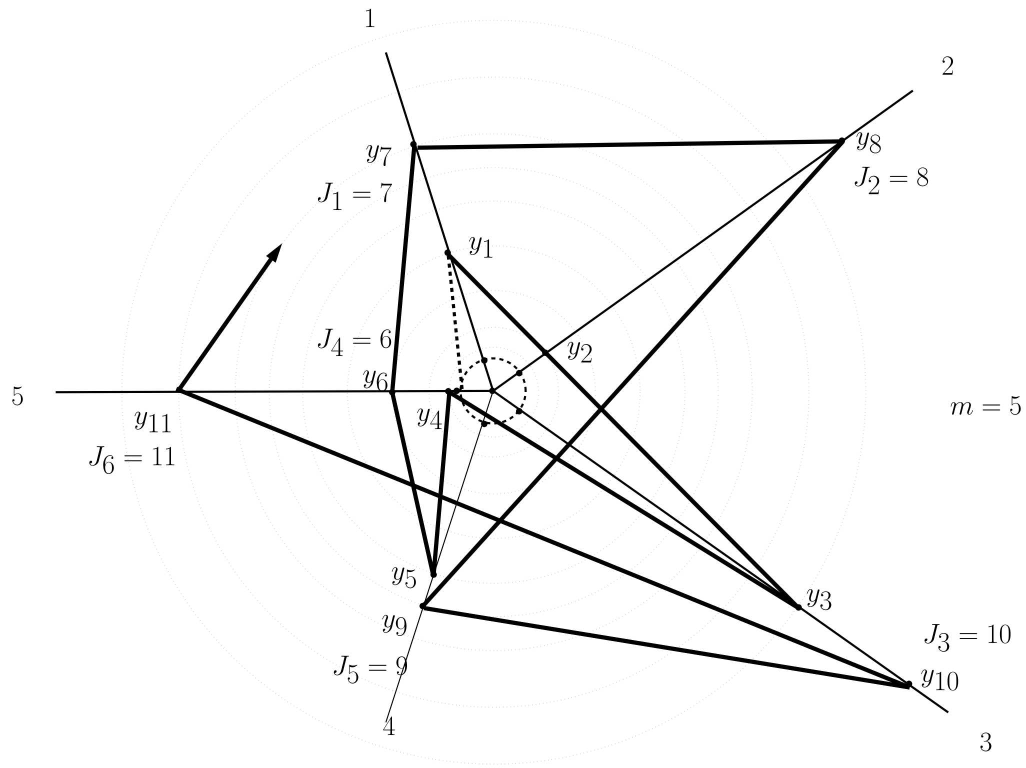


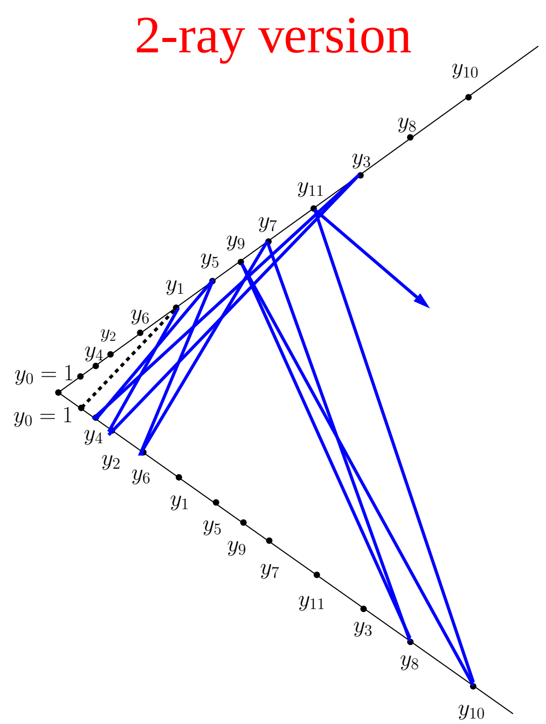
2-ray version



Example: From m toward 2 rays

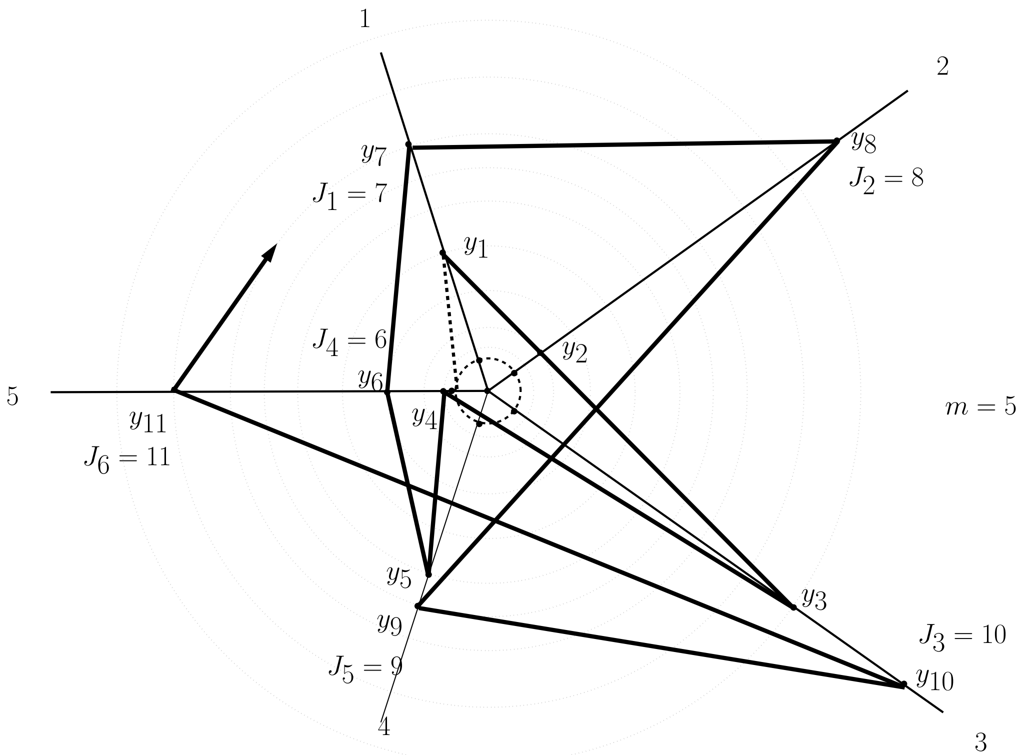
$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, indices J_k original *visiting order*!



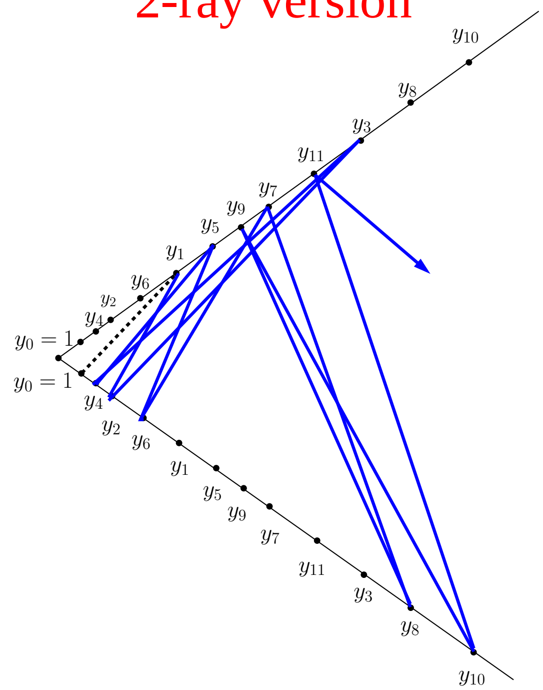


Example: From m toward 2 rays

$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!



2-ray version

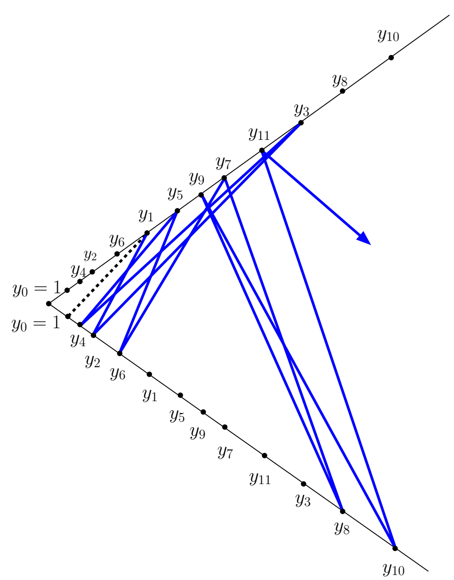


The 2-ray version gives a lower bound on the original version!

Optimize the 2-ray version

Optimize the 2-ray version

$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$ indices J_k original visiting order

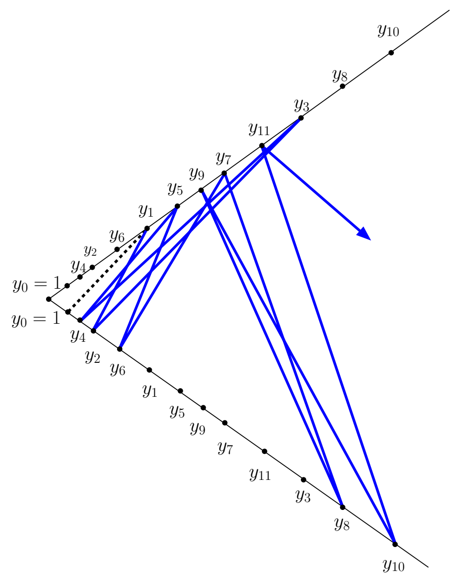


$m = 5$

Optimize the 2-ray version

$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$ indices J_k original visiting order

Choose the best visiting order



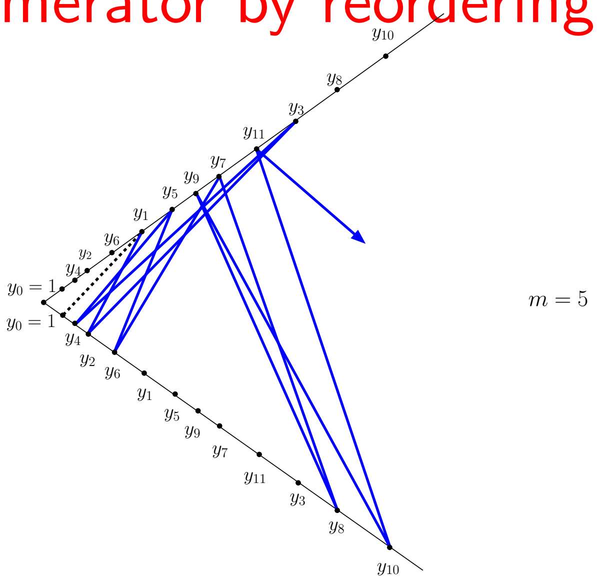
$m = 5$

Optimize the 2-ray version

$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$ indices J_k original visiting order

Choose the best visiting order

Optimize numerator by reordering!

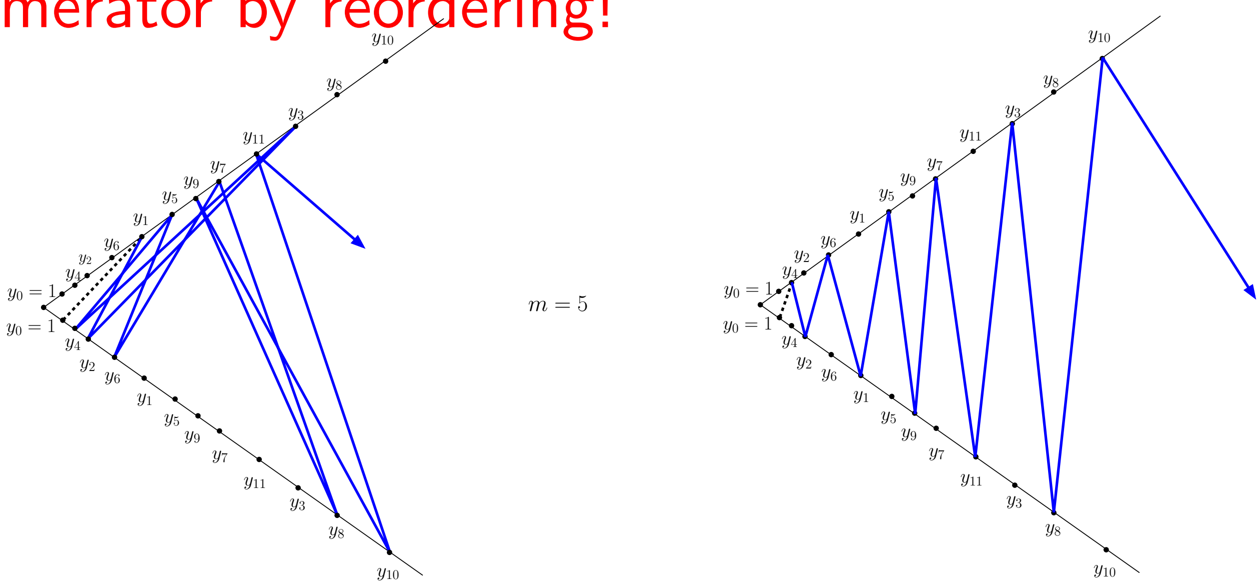


Optimize the 2-ray version

$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$ indices J_k original visiting order

Choose the best visiting order

Optimize numerator by reordering!

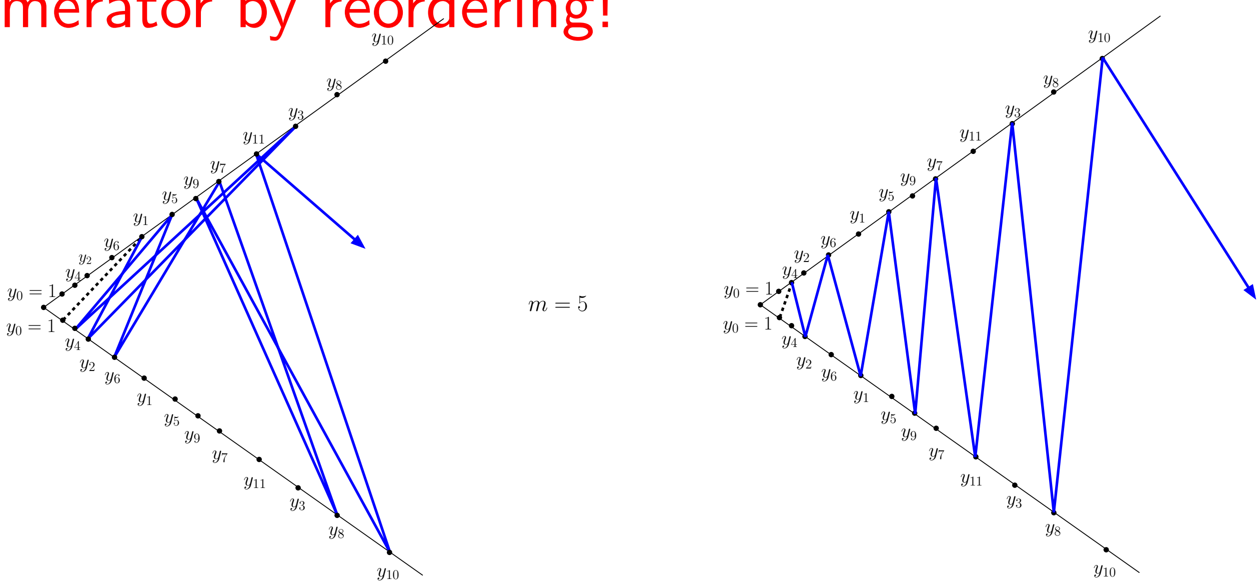


Optimize the 2-ray version

$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_iy_{i+1}\cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$ indices J_k original visiting order

Choose the best visiting order

Optimize numerator by reordering!



Opt. 2-ray vers.: Reordering/optimal visiting order

Lower bound construction

Lower bound construction

$$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k},$$ J_k original *visiting order*!

Lower bound construction

$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2y'_i y'_{i+1} \cos\frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Lower bound construction

$$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2y'_i y'_{i+1} \cos\frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Framework of Gal, discrete m, continuous, unimodality etc.

Lower bound construction

$$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2y'_i y'_{i+1} \cos\frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Framework of Gal, discrete m, continuous, unimodality etc.

Optimized by $y'_i = a^i$, ratio: $f(a,m) = \frac{a^{m-1}}{a-1}\sqrt{1 - 2a\cos\frac{2\pi}{m} + a^2}$

Lower bound construction

$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2y'_i y'_{i+1} \cos\frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Framework of Gal, discrete m, continuous, unimodality etc.

Optimized by $y'_i = a^i$, ratio: $f(a, m) = \frac{a^{m-1}}{a-1}\sqrt{1 - 2a\cos\frac{2\pi}{m} + a^2}$

$m \mapsto \infty$ then $f(a_{\min}, m) \mapsto 17.289\ldots$

Lower bound construction

$$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2y'_i y'_{i+1} \cos\frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Framework of Gal, discrete m, continuous, unimodality etc.

Optimized by $y'_i = a^i$, ratio: $f(a,m) = \frac{a^{m-1}}{a-1}\sqrt{1 - 2a\cos\frac{2\pi}{m} + a^2}$

$m \mapsto \infty$ then $f(a_{\min}, m) \mapsto 17.289\ldots$

Example: $m = 100000$, compute $a_{\min} = 1.000009764\ldots$ and $f(a_{\min}, 100000) = 17.289\ldots$

Conclusion

Conclusion

- Searching for a point in the plane

Conclusion

- Searching for a point in the plane
- Spiral search is optimal

Conclusion

- Searching for a point in the plane
- Spiral search is optimal
- Discrete m-ray version, 2-ray version for m

Conclusion

- Searching for a point in the plane
- Spiral search is optimal
- Discrete m-ray version, 2-ray version for m
- Find a functional that gives a lower bound

Conclusion

- Searching for a point in the plane
- Spiral search is optimal
- Discrete m-ray version, 2-ray version for m
- Find a functional that gives a lower bound
- Spiral conjecture of similar settings