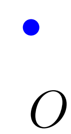
On the optimality of spiral search

Elmar Langetepe
University of Bonn

Searching for a point in the plane

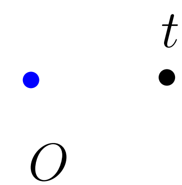
Searching for a point in the plane

- Startpoint O,



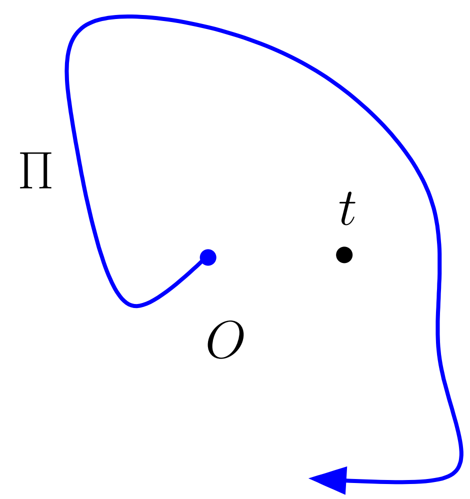
Searching for a point in the plane

- Startpoint O, unknown target t



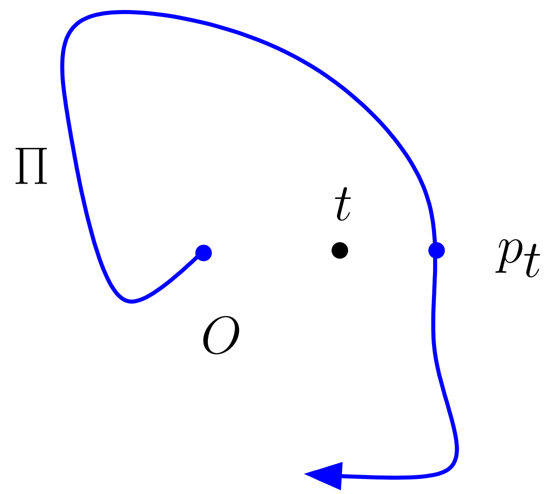
Searching for a point in the plane

- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane



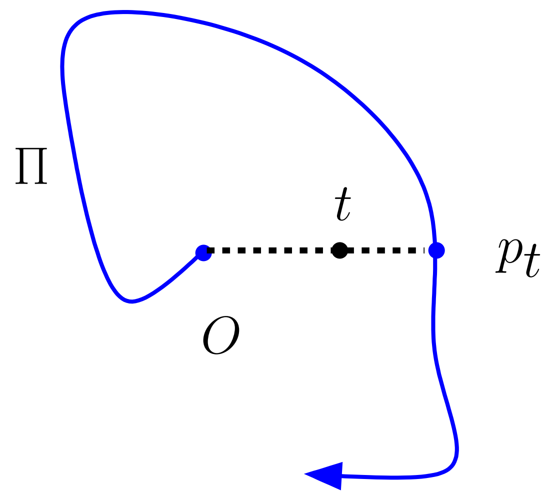
Searching for a point in the plane

- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane
- Find t if $t \in Op_t$



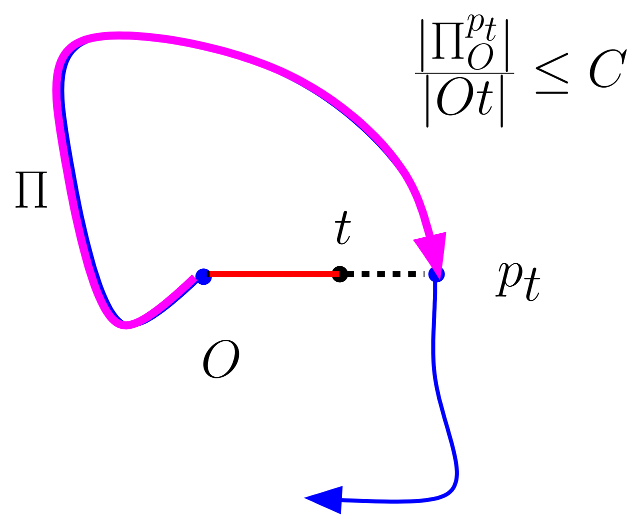
Searching for a point in the plane

- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane
- Find t if $t \in Op_t$



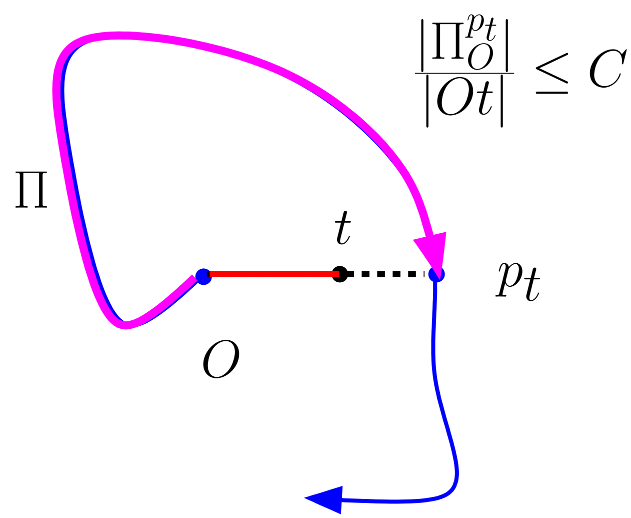
Searching for a point in the plane

- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane
- Find t if $t \in Op_t$
- Competitive ratio: worst-case $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$



Searching for a point in the plane

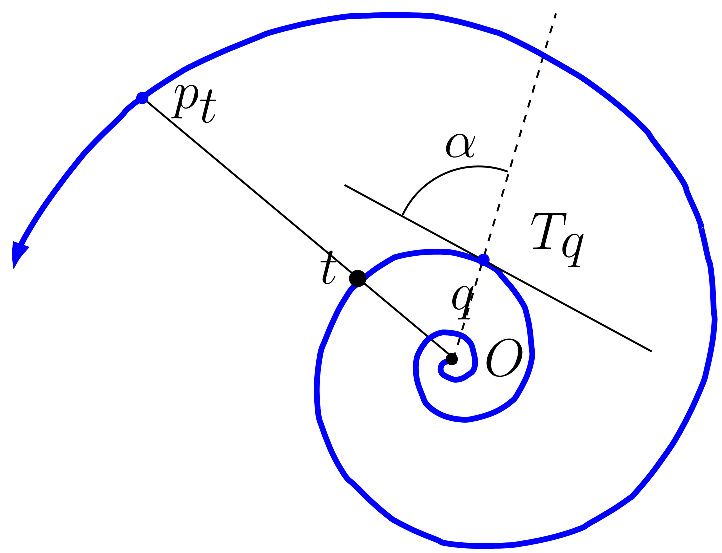
- Startpoint O, unknown target t
- Strategy Π *sweeps* the plane
- Find t if $t \in Op_t$
- Competitive ratio: worst-case
  $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$
- Gal 1980!



Spiral search

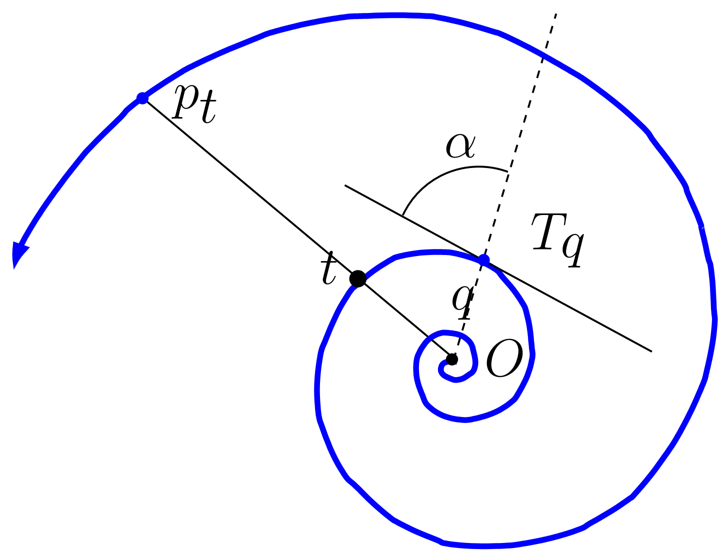
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$



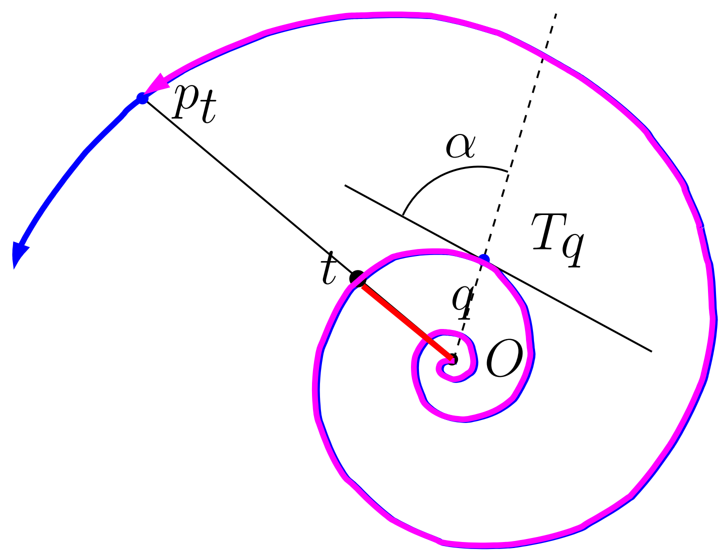
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target!



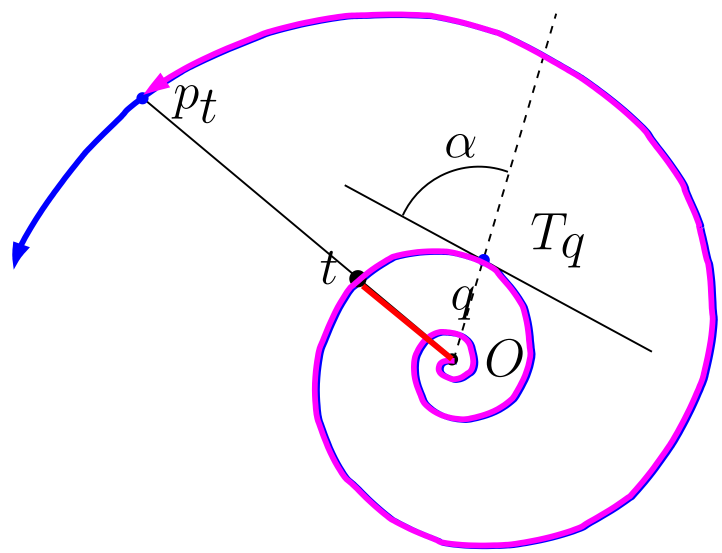
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target!
- $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$, $C = 17.289\ldots$



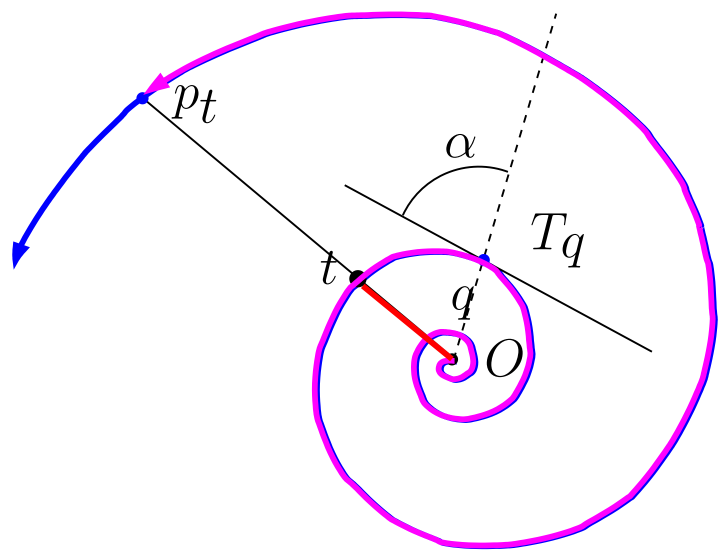
Spiral search

- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target!
- $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$, $C = 17.289\ldots$
- Best periodic and monotone strategy, optimal?



Spiral search

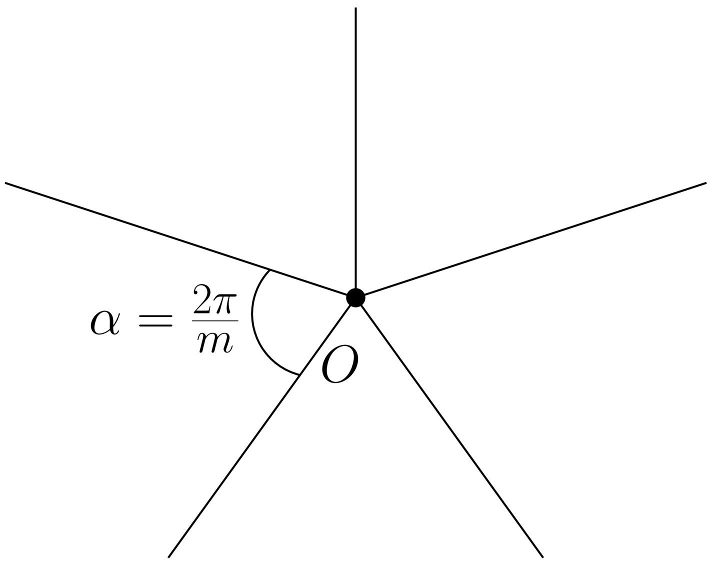
- Spiral search! $(\varphi, e^{\varphi \cot(\alpha)})$
- Worst Case! Sligthly miss the target!
- $C := \sup_t \frac{|\Pi_O^{p_t}|}{|Ot|}$, $C = 17.289\ldots$
- Best periodic and monotone strategy, optimal?
- Radius vector $X(\theta)$: θ increases and $X(\theta + 2\pi) \geq X(\theta)$



Lower bound construction

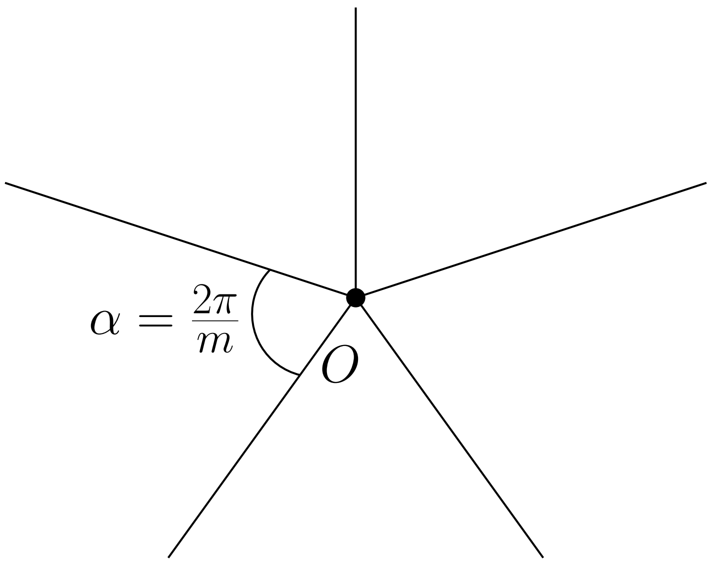
Lower bound construction

- Bundle of m rays



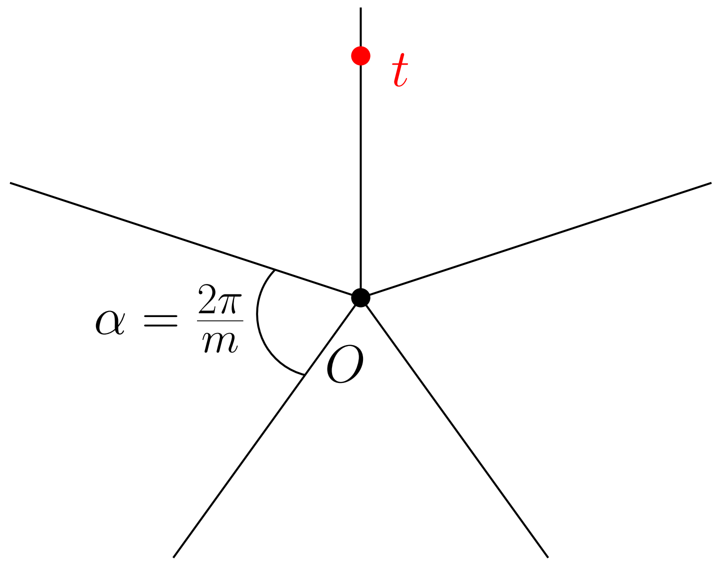
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$



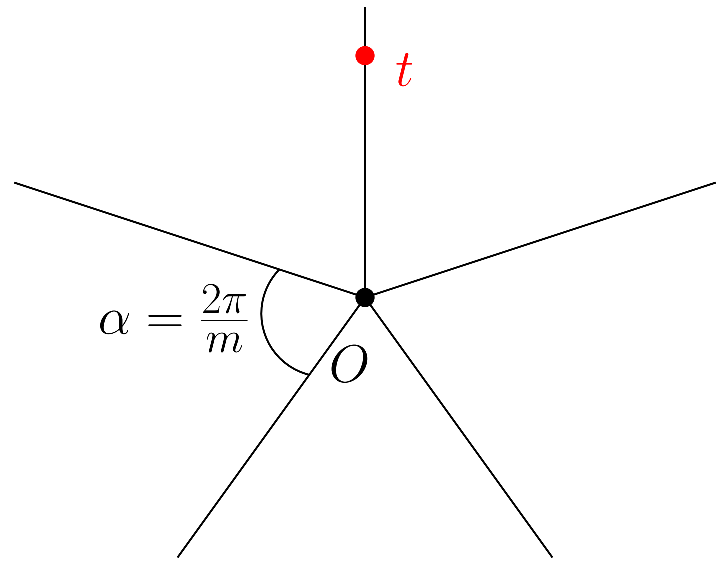
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray



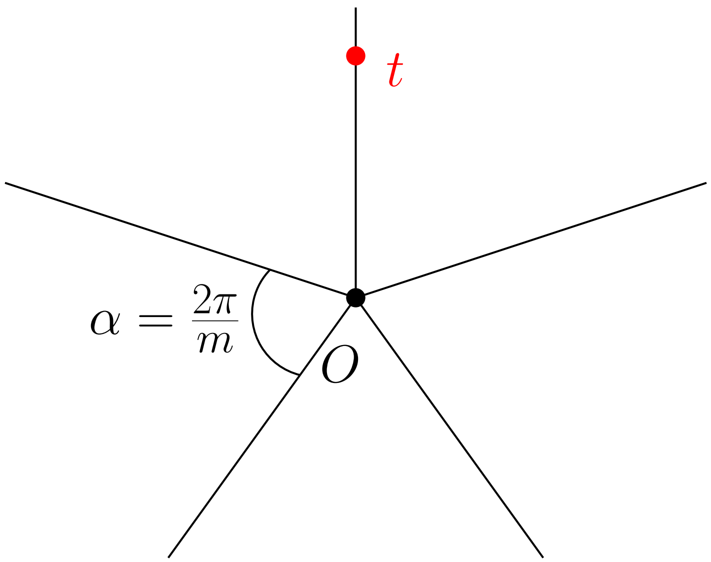
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end



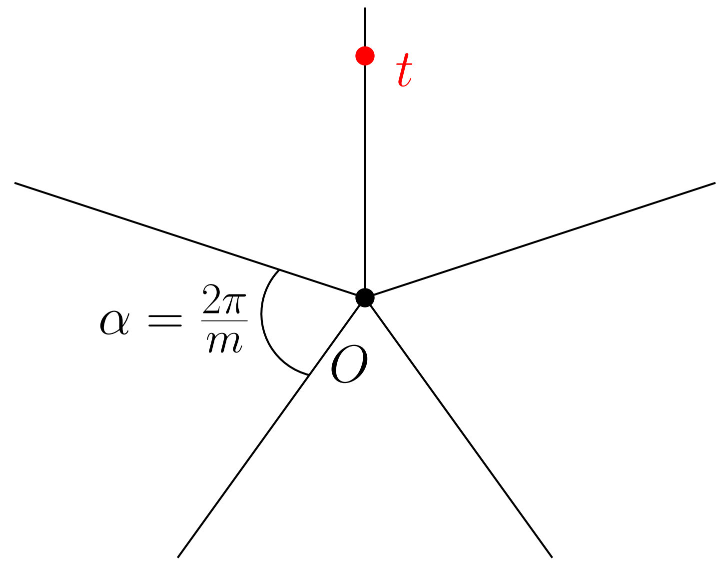
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies



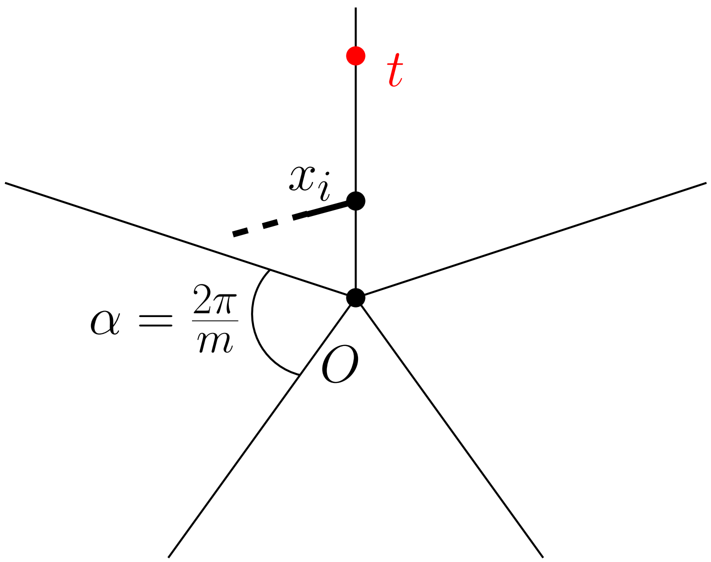
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order



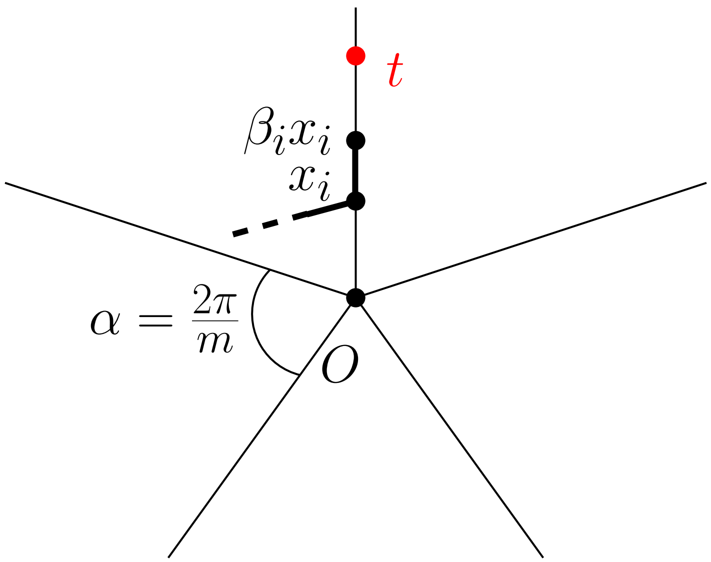
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)



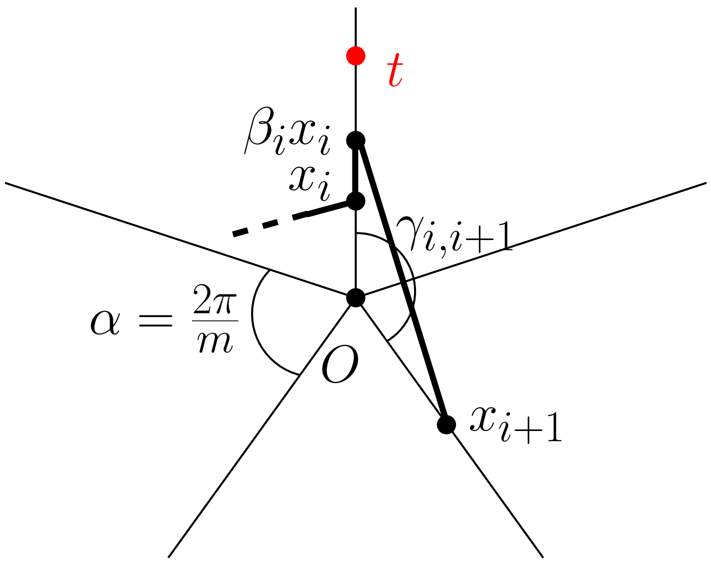
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)



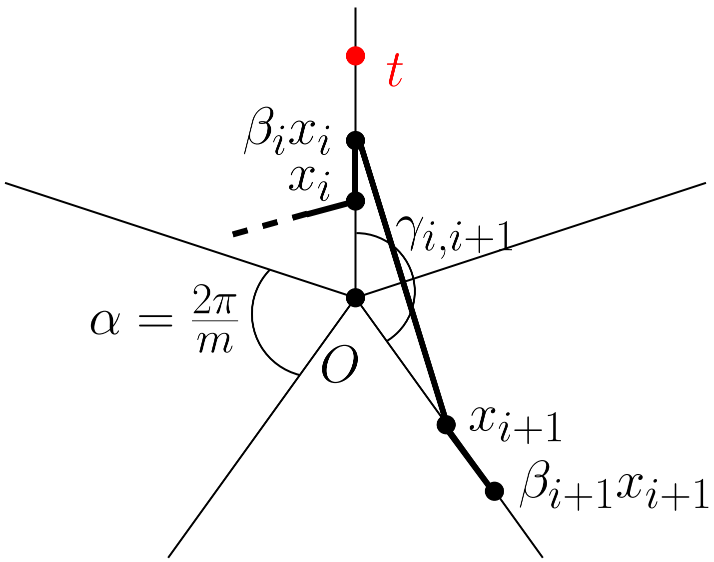
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)



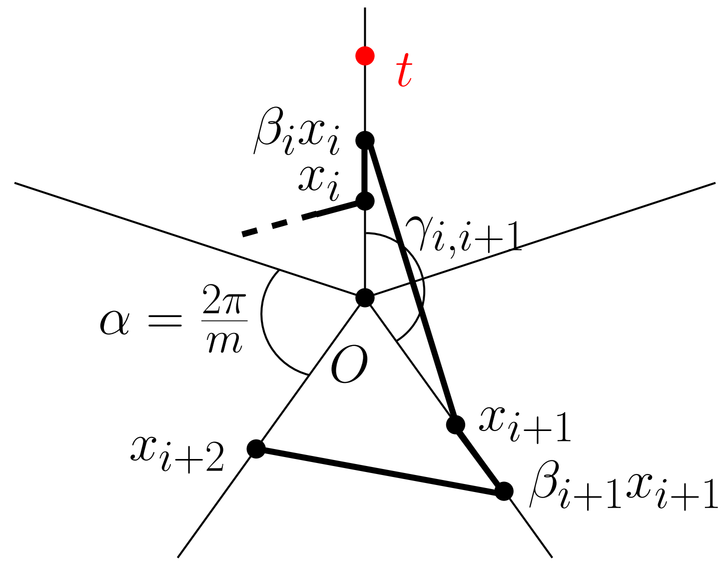
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)



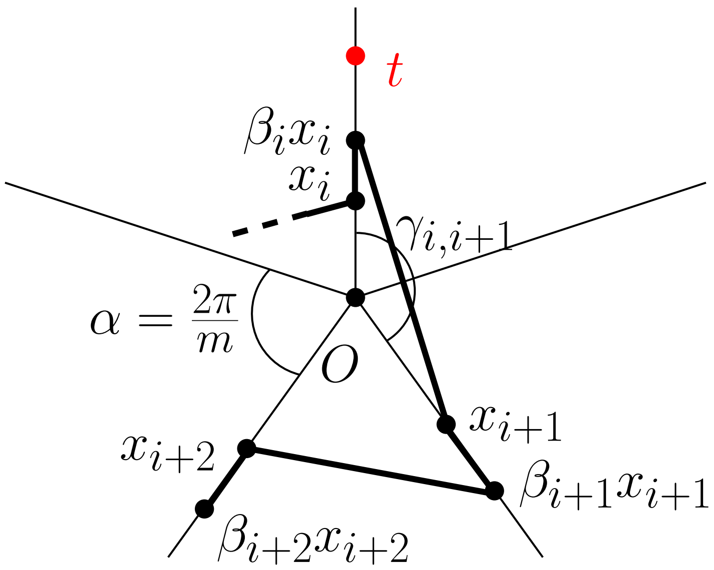
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)



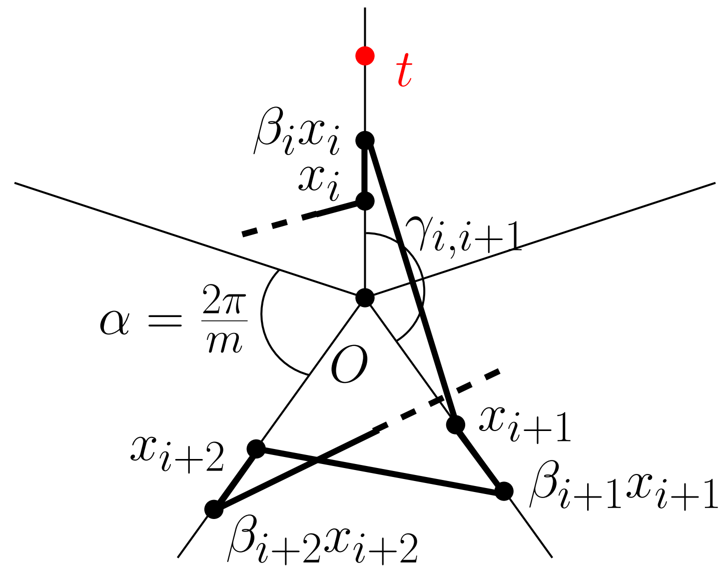
Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)

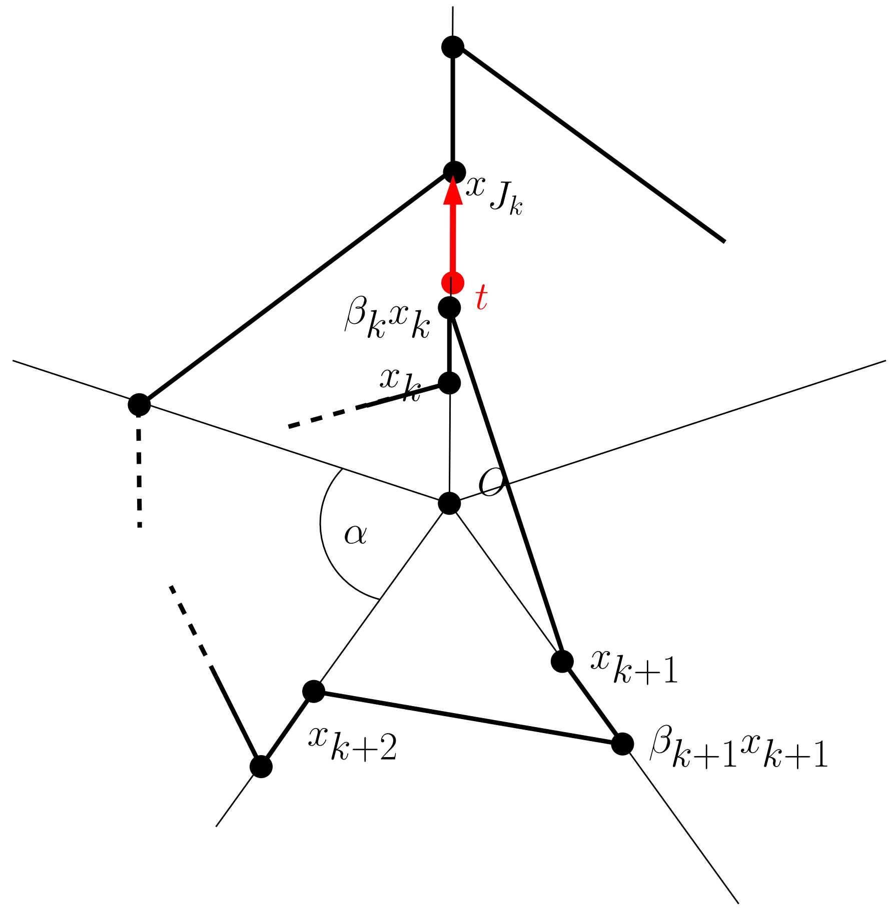


Lower bound construction

- Bundle of m rays
- Separated by an angle $\alpha = \frac{2\pi}{m}$
- Target is on a ray
- Let m go to infinity at the end
- Also non-periodic and non-monotone strategies
- Stragegy S: Visits the rays in some order
- Hit x_i, leave $\beta_i x_i$ ($\beta_i \geq 1$)

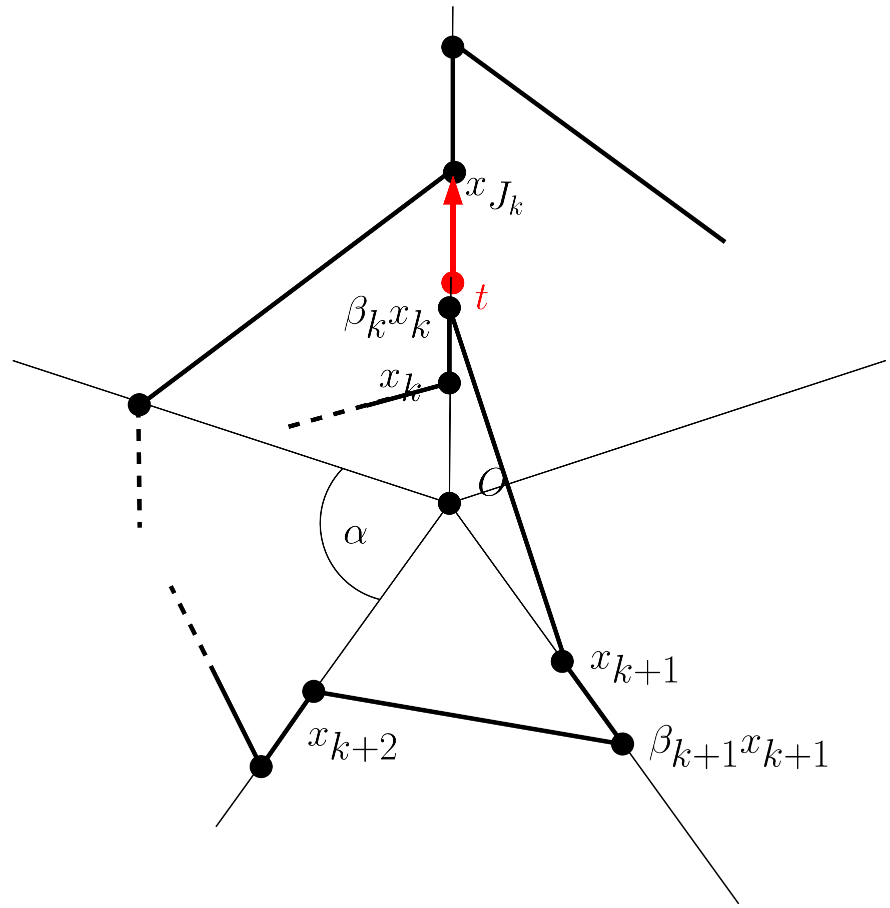


Lower bound construction: m rays



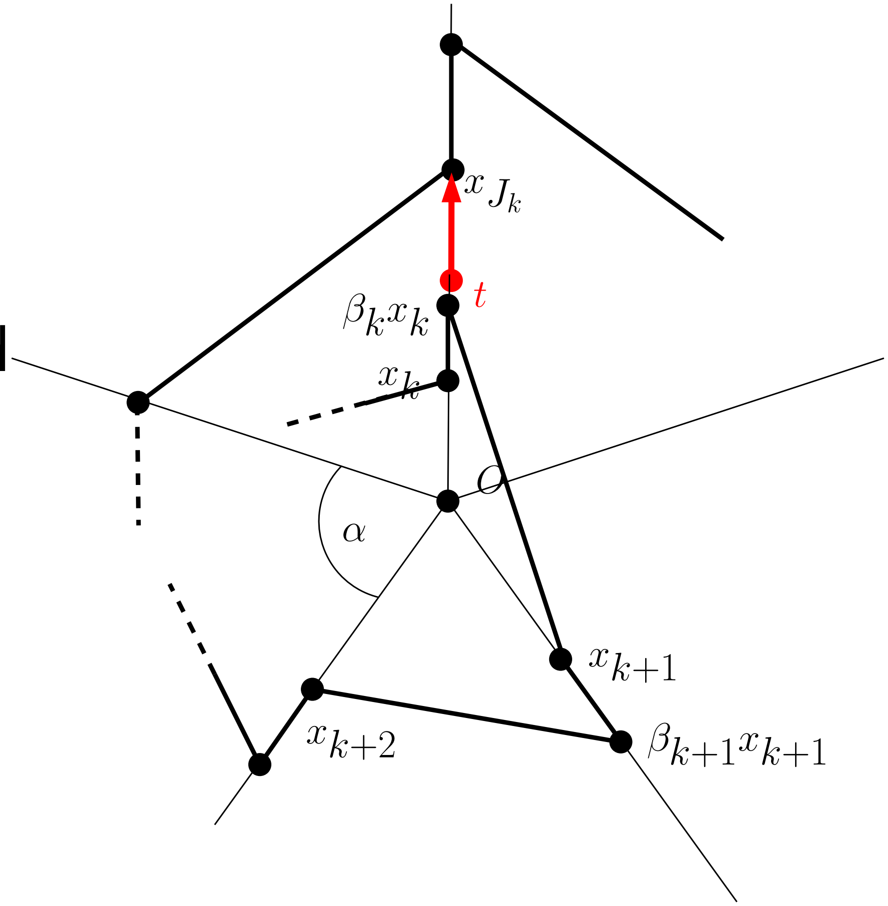
Lower bound construction: m rays

- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$



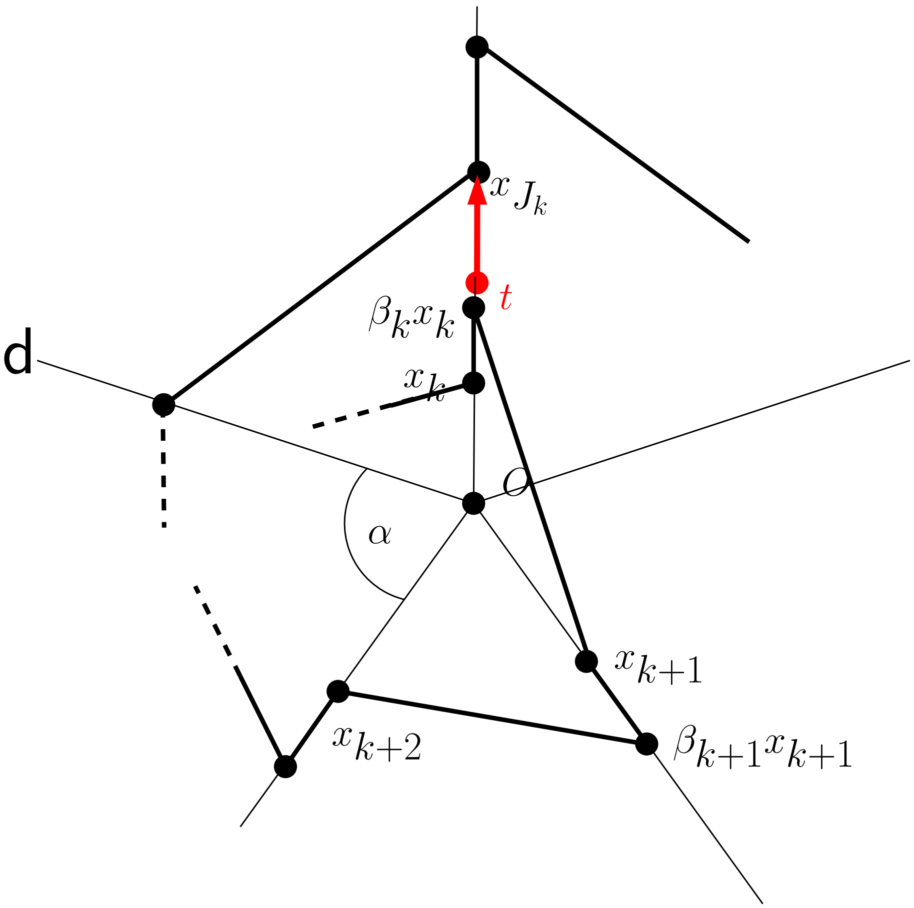
Lower bound construction: m rays

- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$
- Find t on a ray at step x_{J_k} visited at step k at the last time



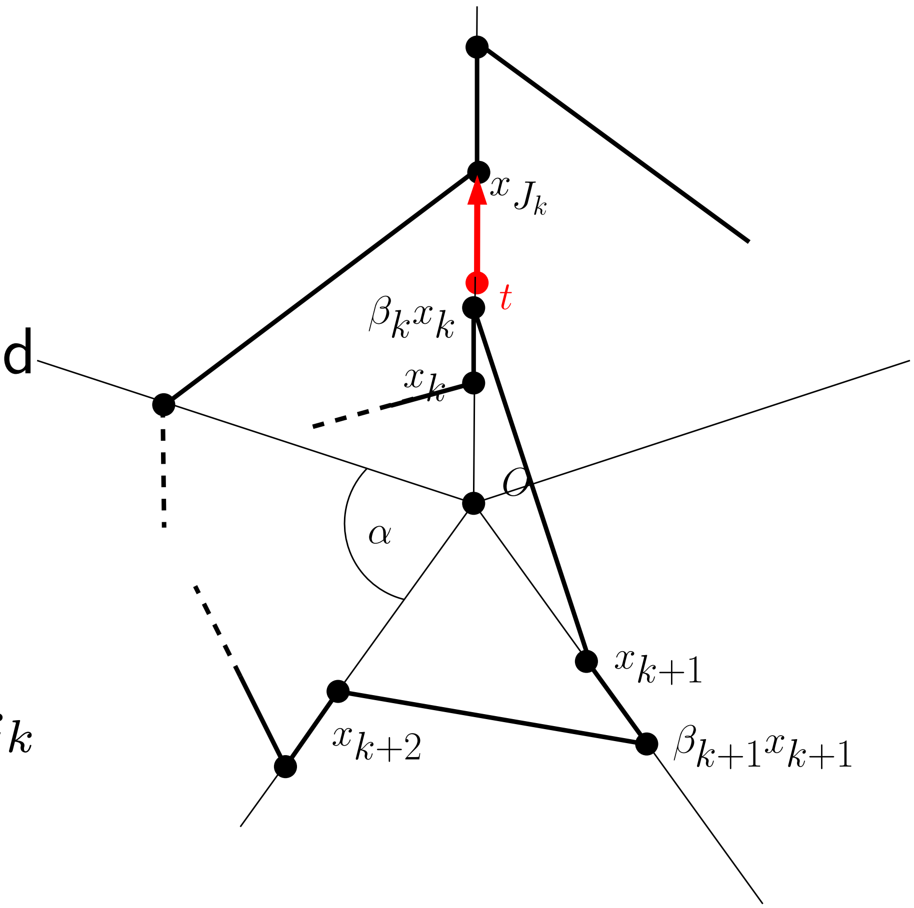
Lower bound construction: m rays

- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$
- Find t on a ray at step x_{J_k} visited at step k at the last time
- Any order is possible: $i = J_k$, k?



Lower bound construction: m rays

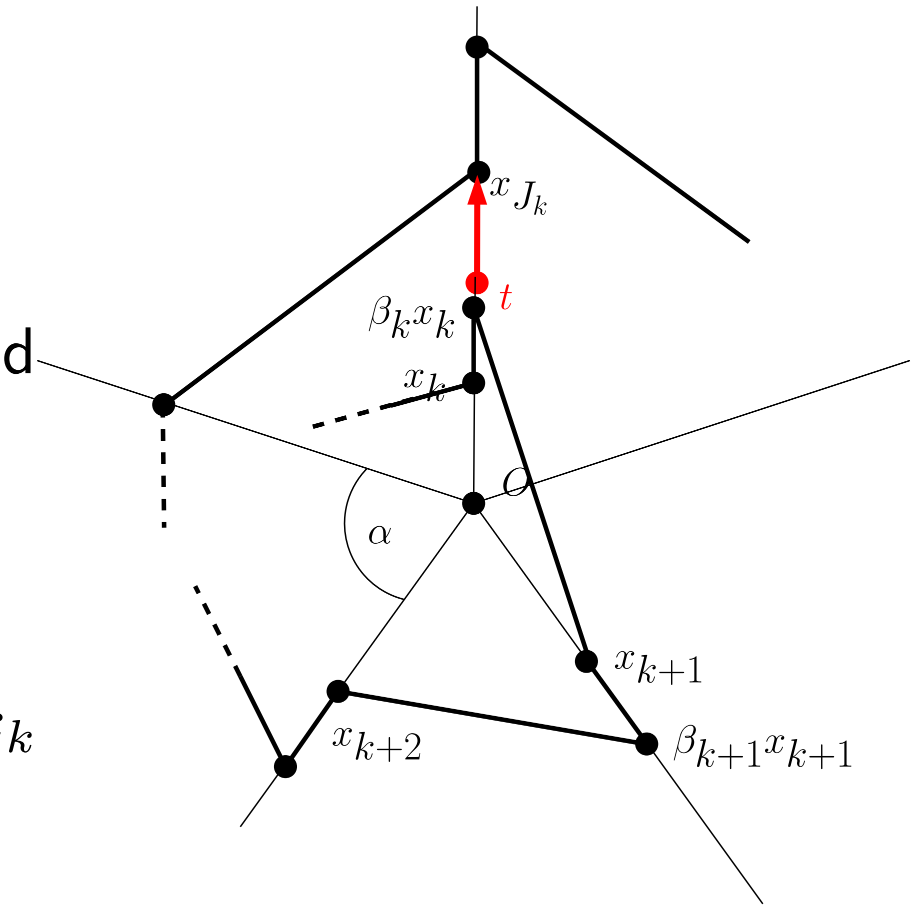
- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$
- Find t on a ray at step x_{J_k} visited at step k at the last time
- Any order is possible: $i = J_k$, k?
- Worst-case for k: t close to $\beta_k x_k$



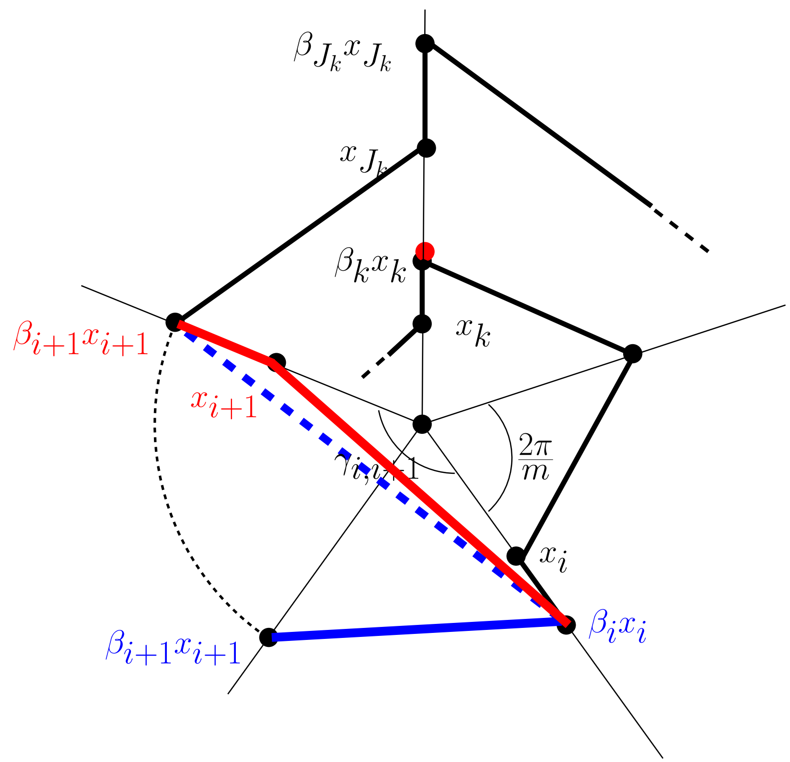
Lower bound construction: m rays

- $S = (x_1, \beta_1 x_1, x_2, \beta_2 x_2, \ldots)$
- Find t on a ray at step x_{J_k} visited at step k at the last time
- Any order is possible: $i = J_k,\ k$?
- Worst-case for k: t close to $\beta_k x_k$
- Ratio: $C(S) = \sup_k$

$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)}{\beta_k x_k}$$



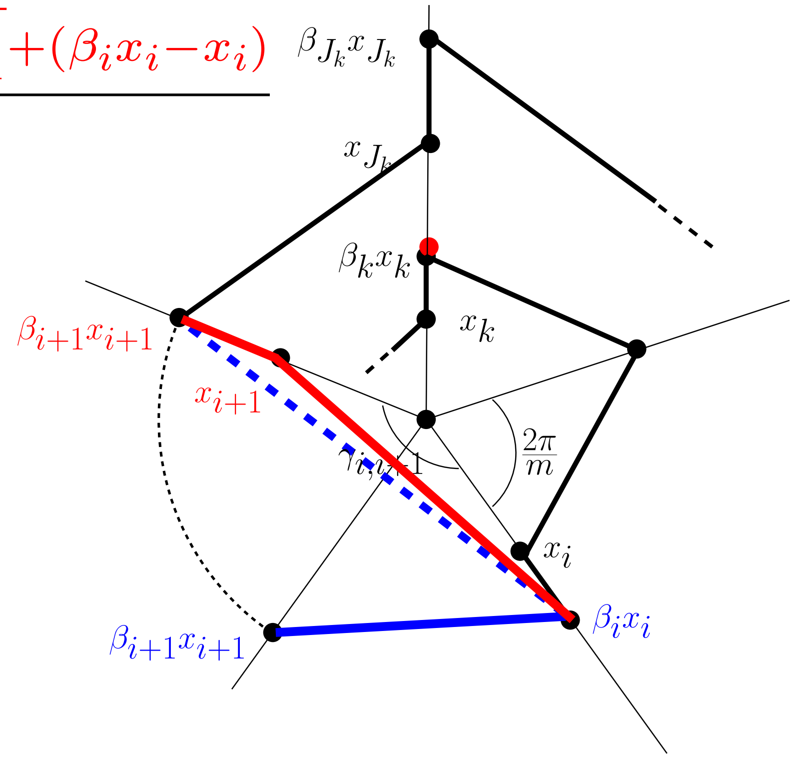
Lower bound construction: m rays



Lower bound construction: m rays

- Ratio: $C(S) = \sup_k$

$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos\gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)}{\beta_k x_k}$$



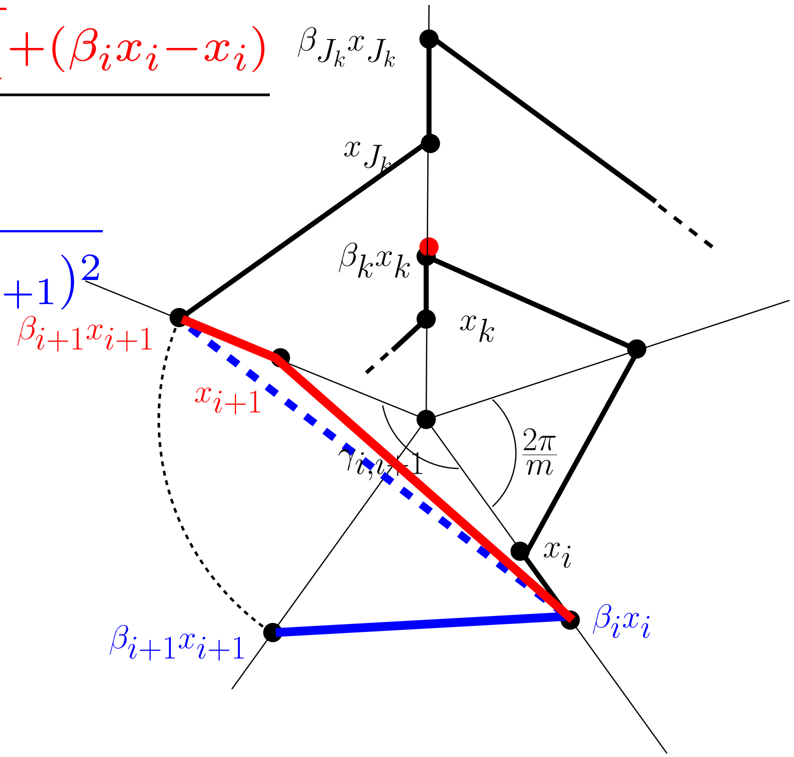
Lower bound construction: m rays

- Ratio: $C(S) = \sup_k$

$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)}{\beta_k x_k}$$

- Shrinking numerator:

$$\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i \beta_{i+1} x_{i+1} \cos \frac{2\pi}{m} + (\beta_{i+1} x_{i+1})^2}$$



Lower bound construction: m rays

- Ratio: $C(S) = \sup_k$

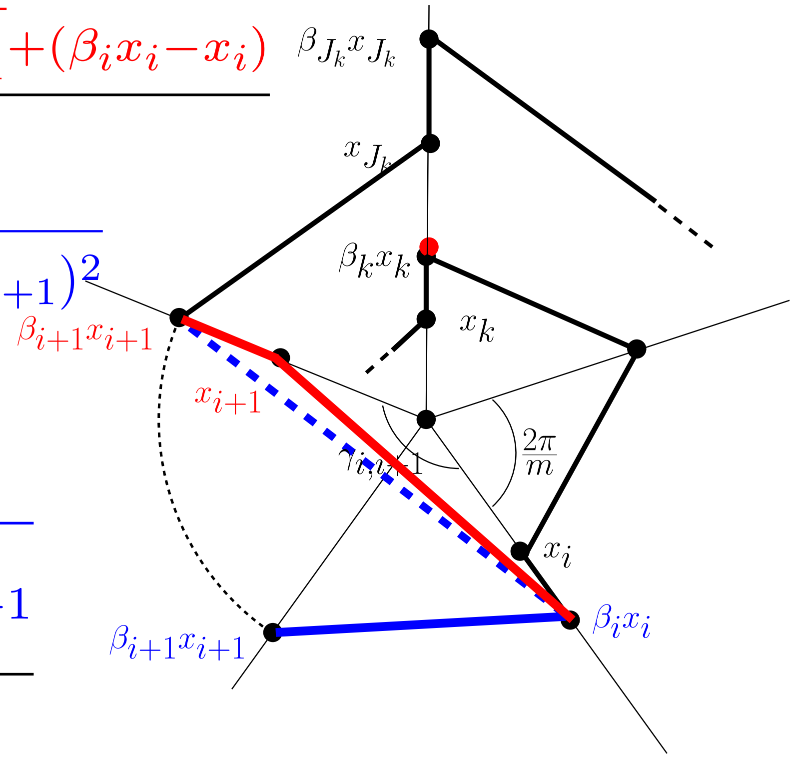
$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos\gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)}{\beta_k x_k}$$

- Shrinking numerator:

$$\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i \beta_{i+1} x_{i+1} \cos\frac{2\pi}{m} + (\beta_{i+1} x_{i+1})^2}$$

- Lower bound ($y_i := \beta_i x_i$): $C(S) \geq$

$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$



Lower bound construction: m rays

- Ratio: $C(S) = \sup_k$

$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos\gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)}{\beta_k x_k}$$

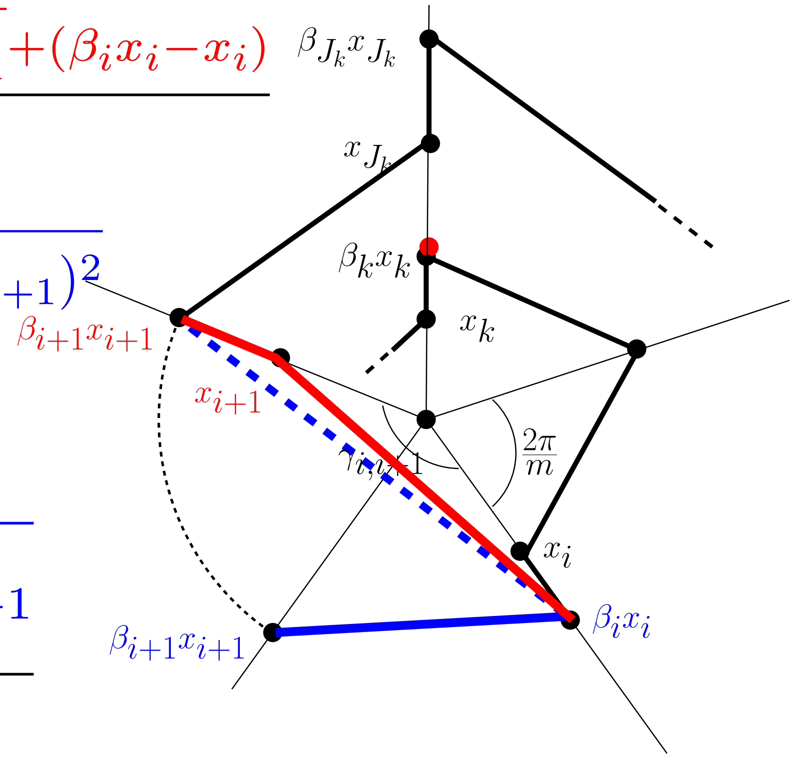
- Shrinking numerator:

$$\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i \beta_{i+1} x_{i+1} \cos\frac{2\pi}{m} + (\beta_{i+1} x_{i+1})^2}$$

- Lower bound ($y_i := \beta_i x_i$): $C(S) \geq$

$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$

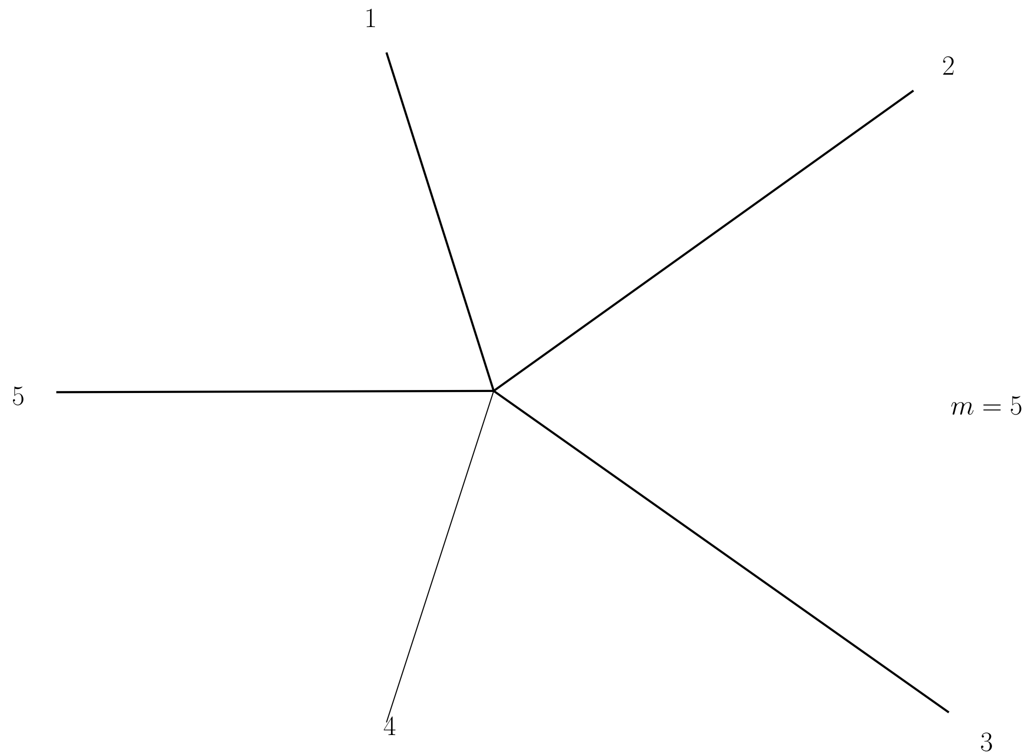
Minimize!!



Lower bound construction: m *imaginary* rays

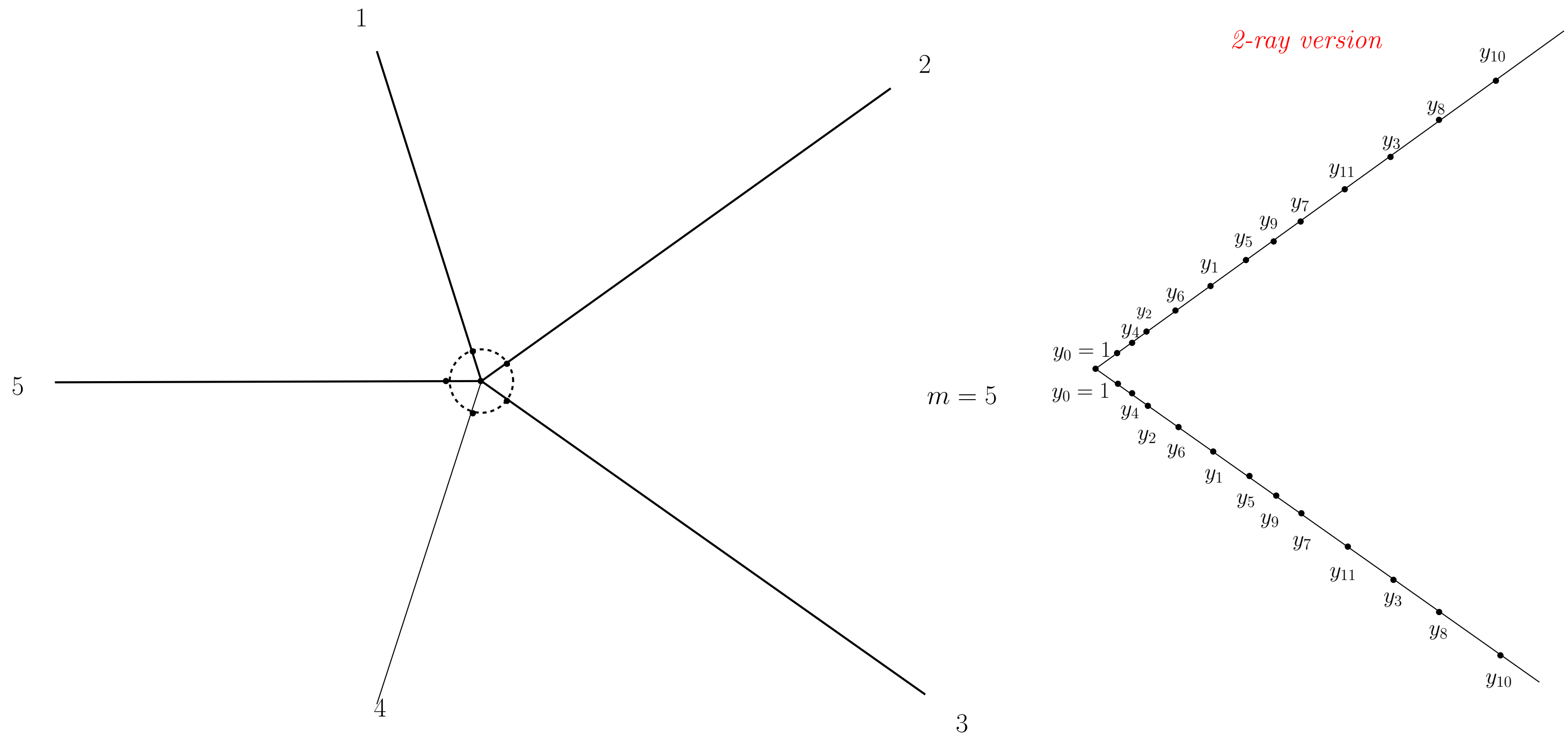
Lower bound construction: m *imaginary* rays

$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!



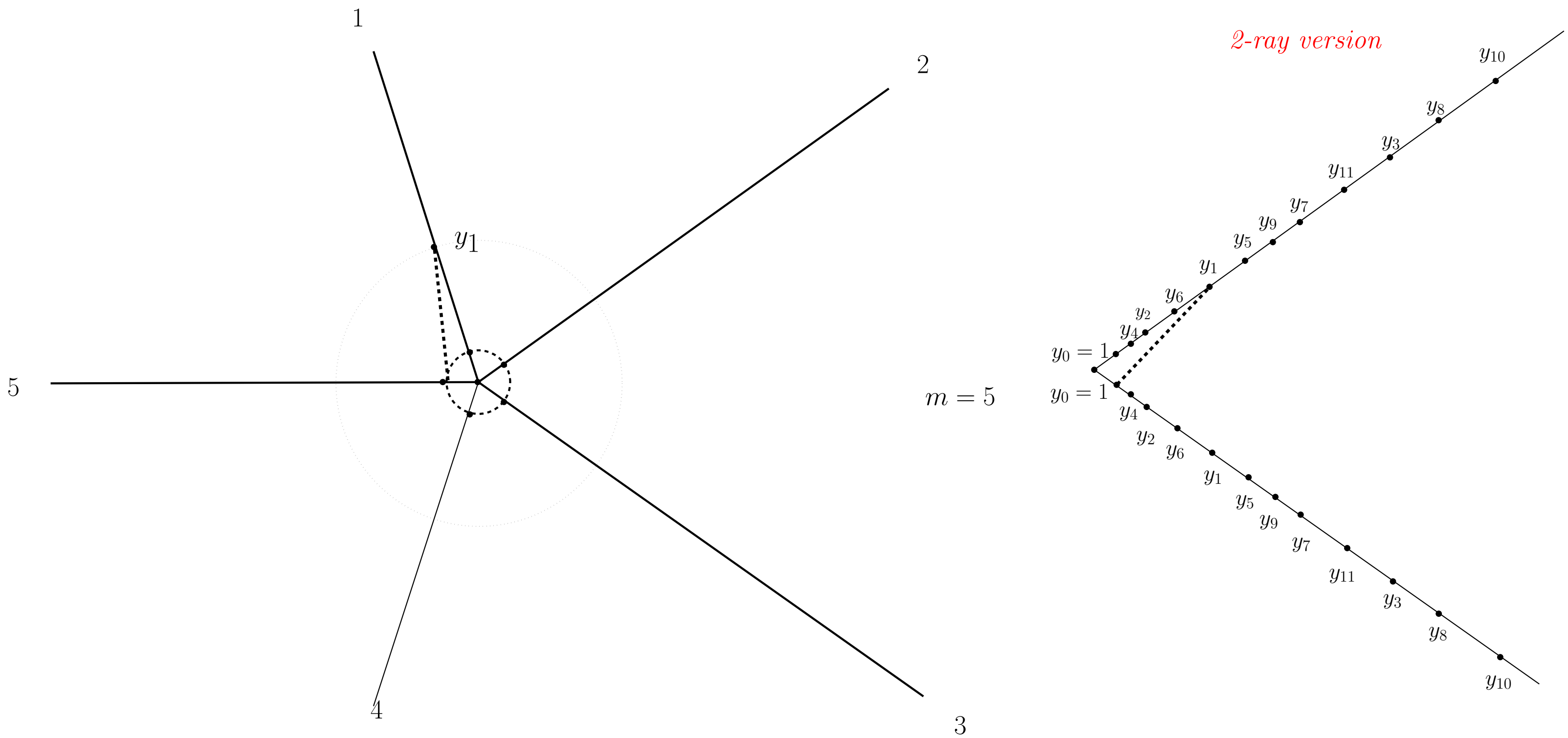
Lower bound construction: m *imaginary* rays

$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!



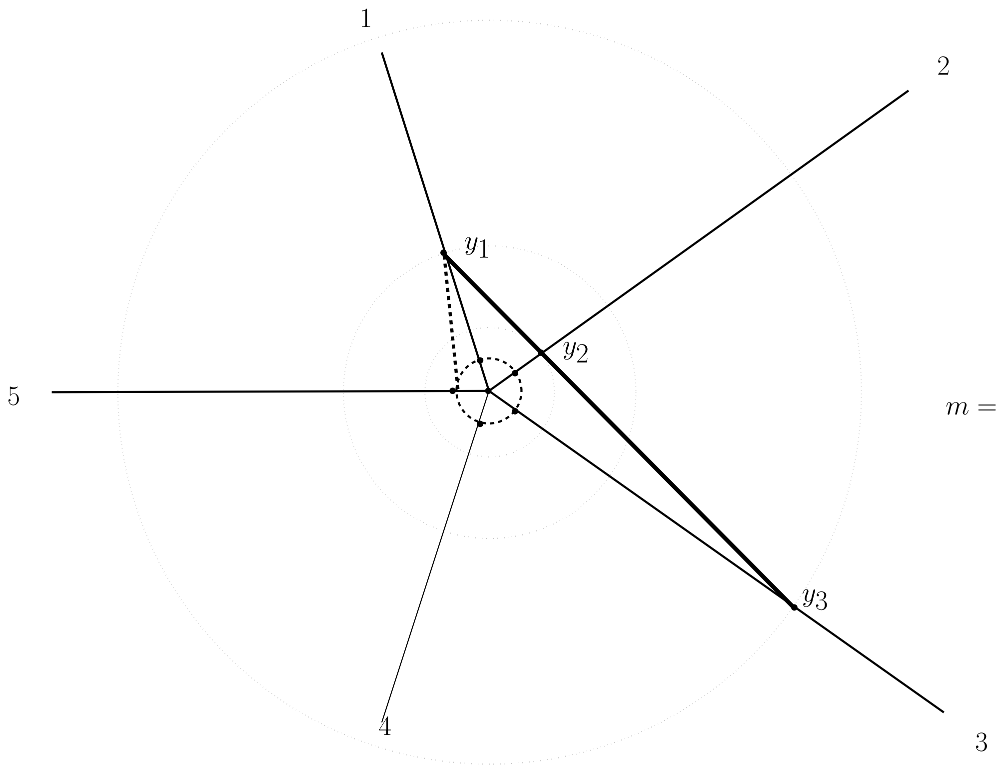
Lower bound construction: m *imaginary* rays

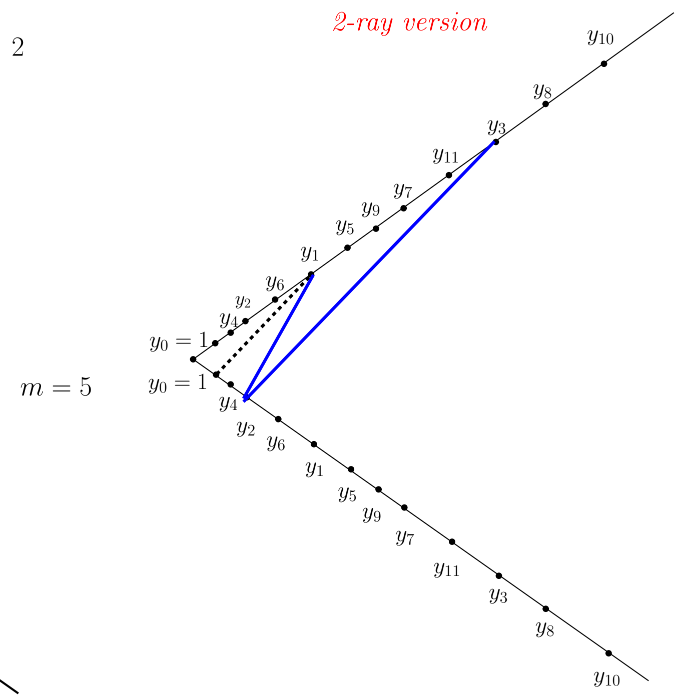
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!



Lower bound construction: m *imaginary* rays

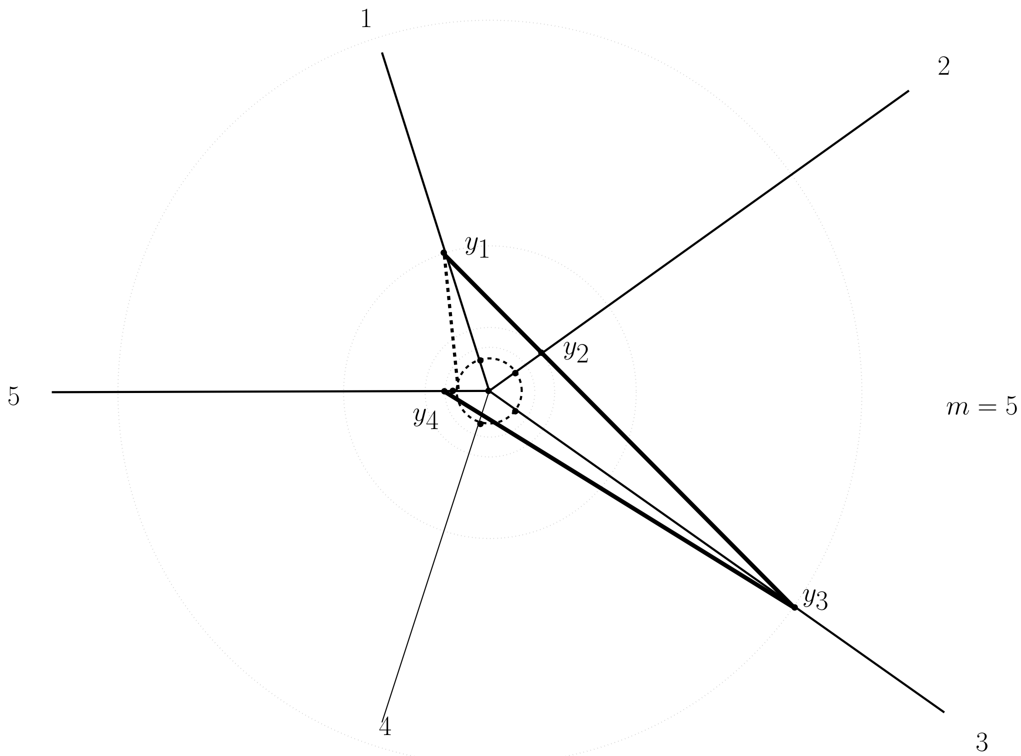
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_iy_{i+1}\cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

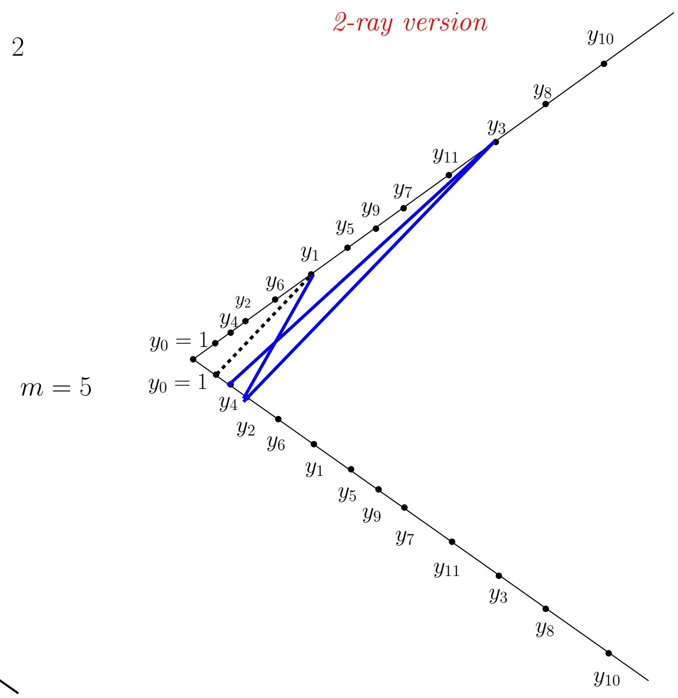




Lower bound construction: m *imaginary* rays

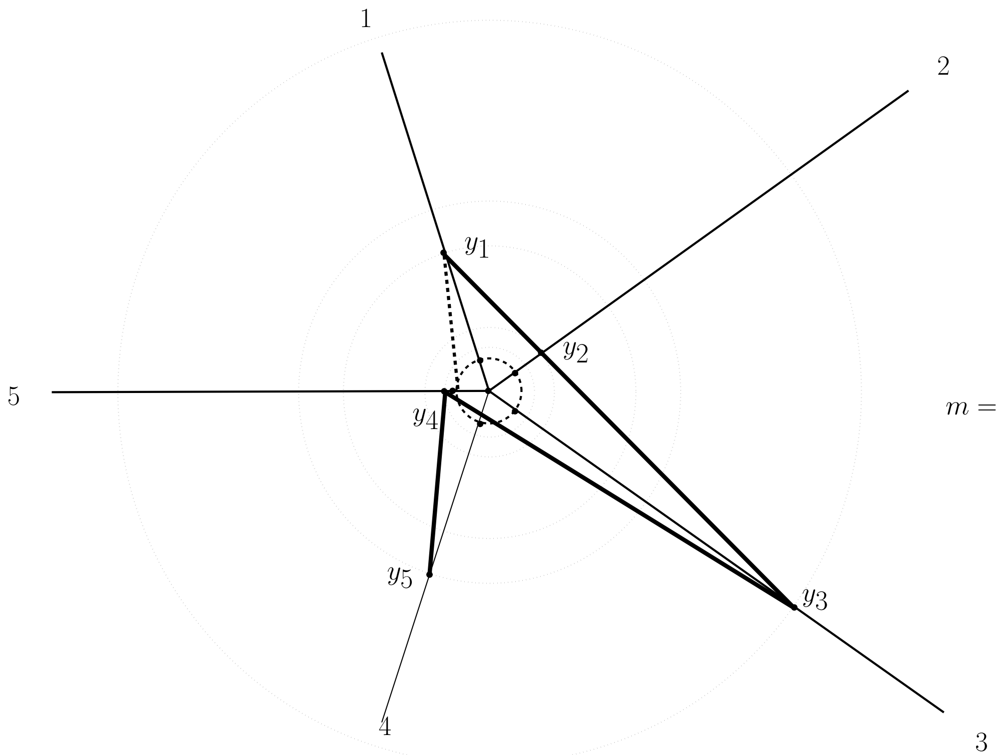
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_iy_{i+1}\cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

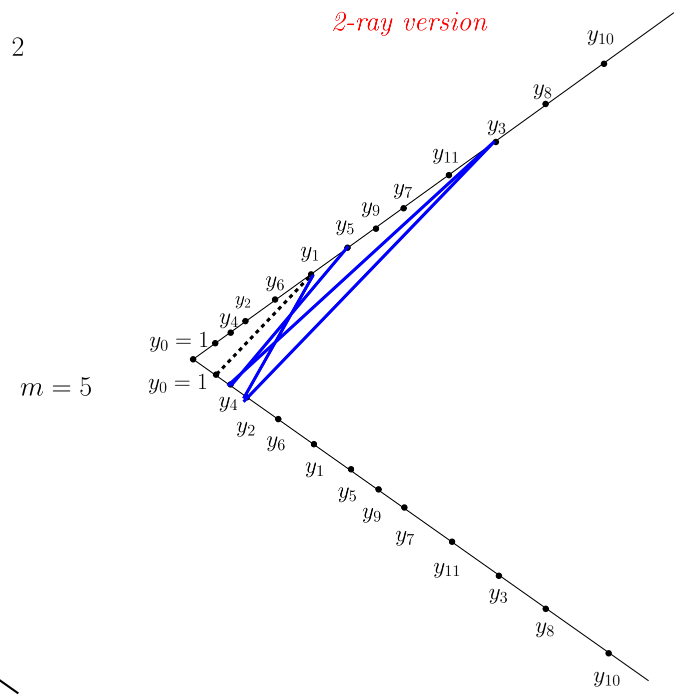




Lower bound construction: m ***imaginary*** rays

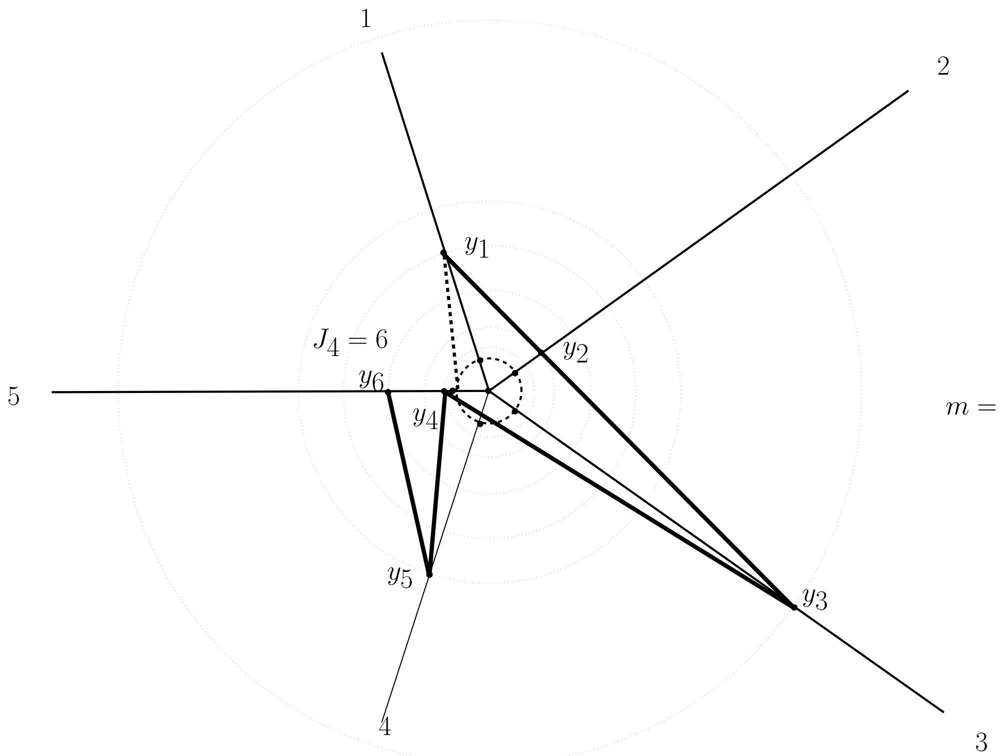
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

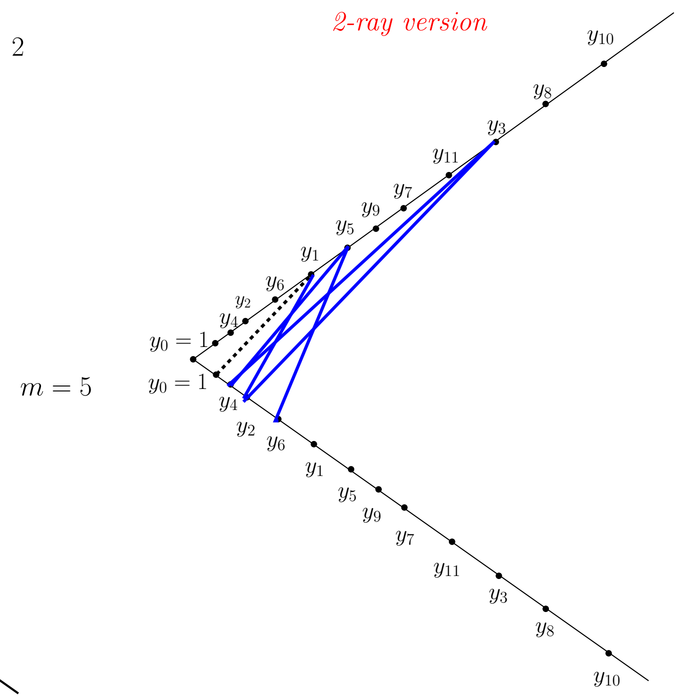




Lower bound construction: m *imaginary* rays

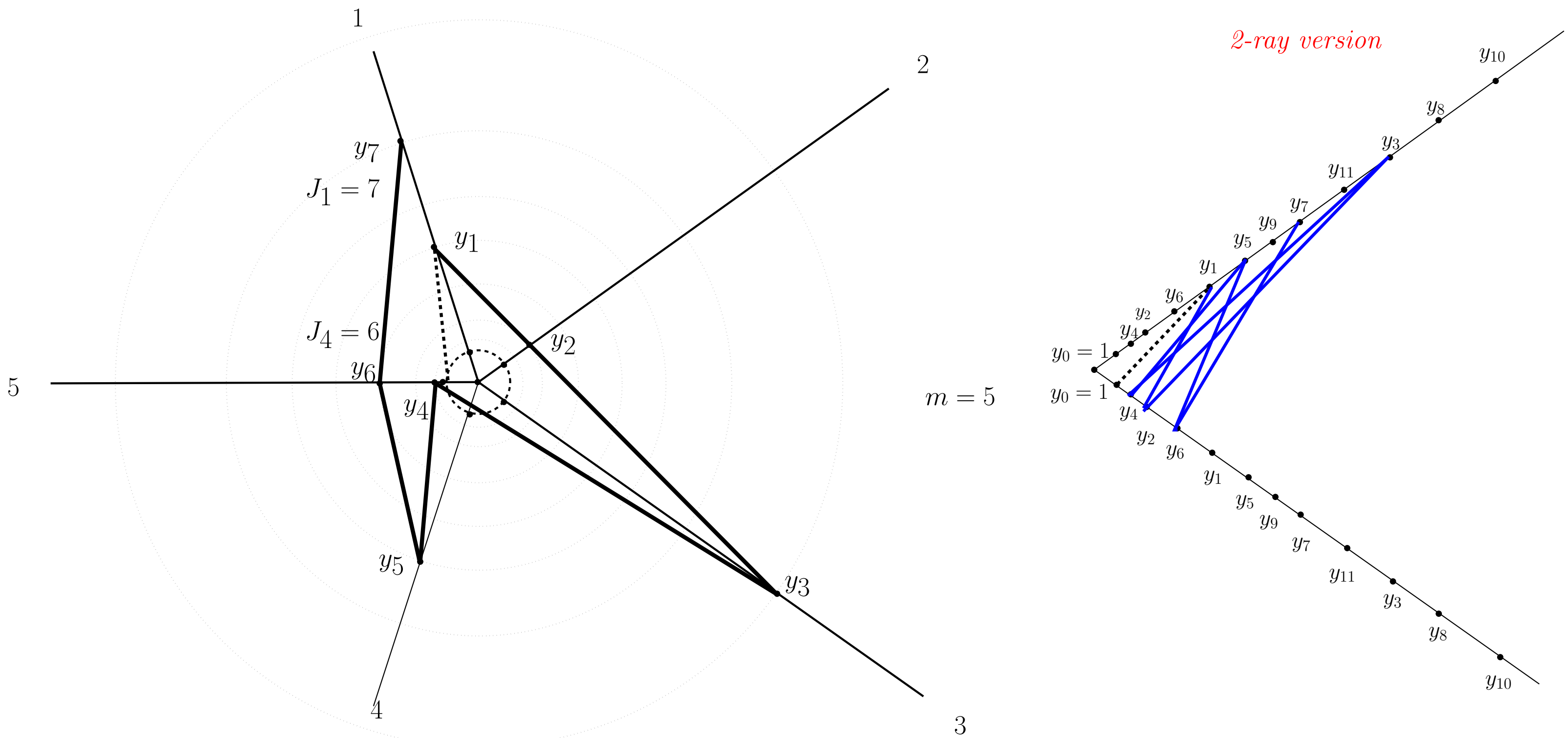
$\sup_k \frac{\sum_{i=1}^{J_k - 2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!





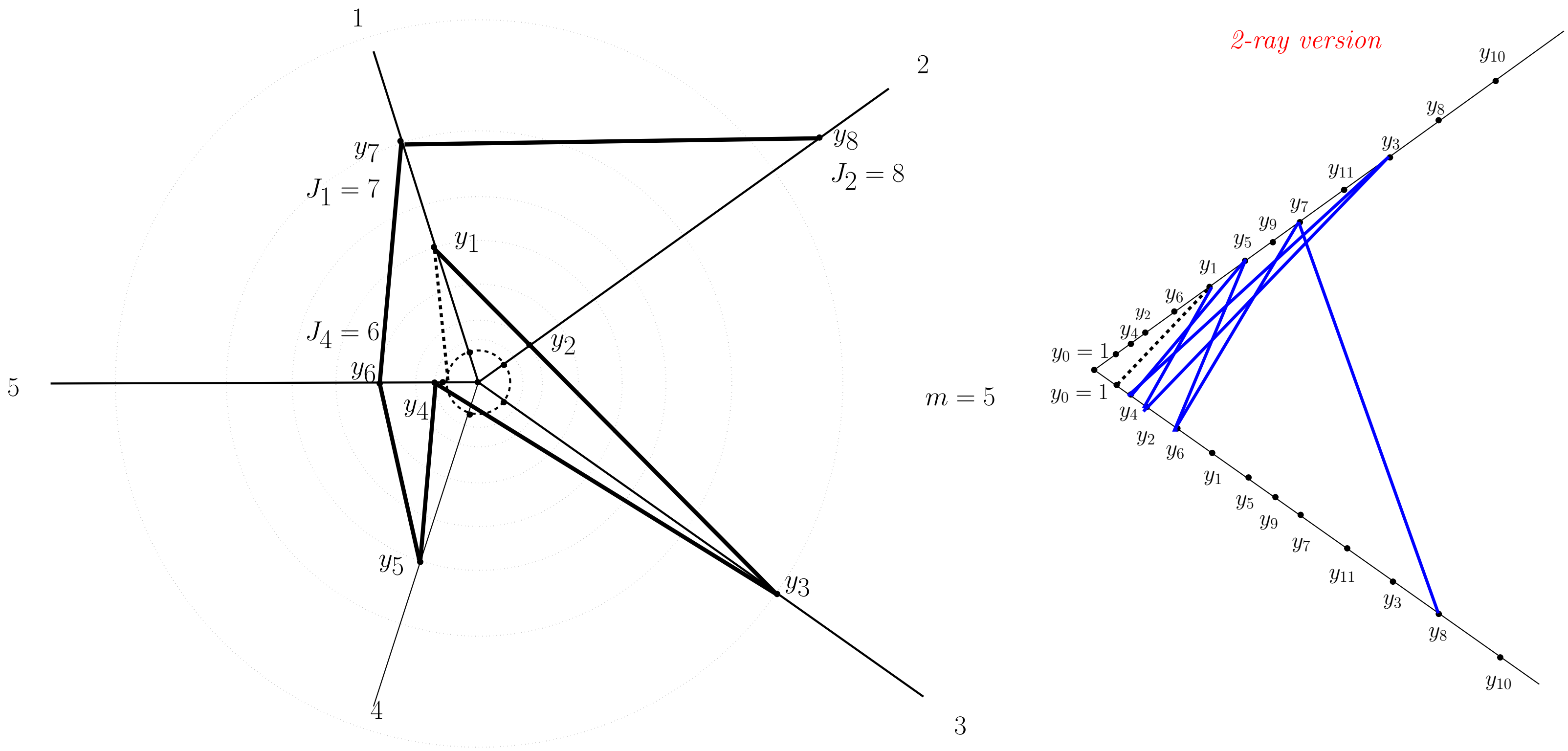
Lower bound construction: m *imaginary* rays

$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!



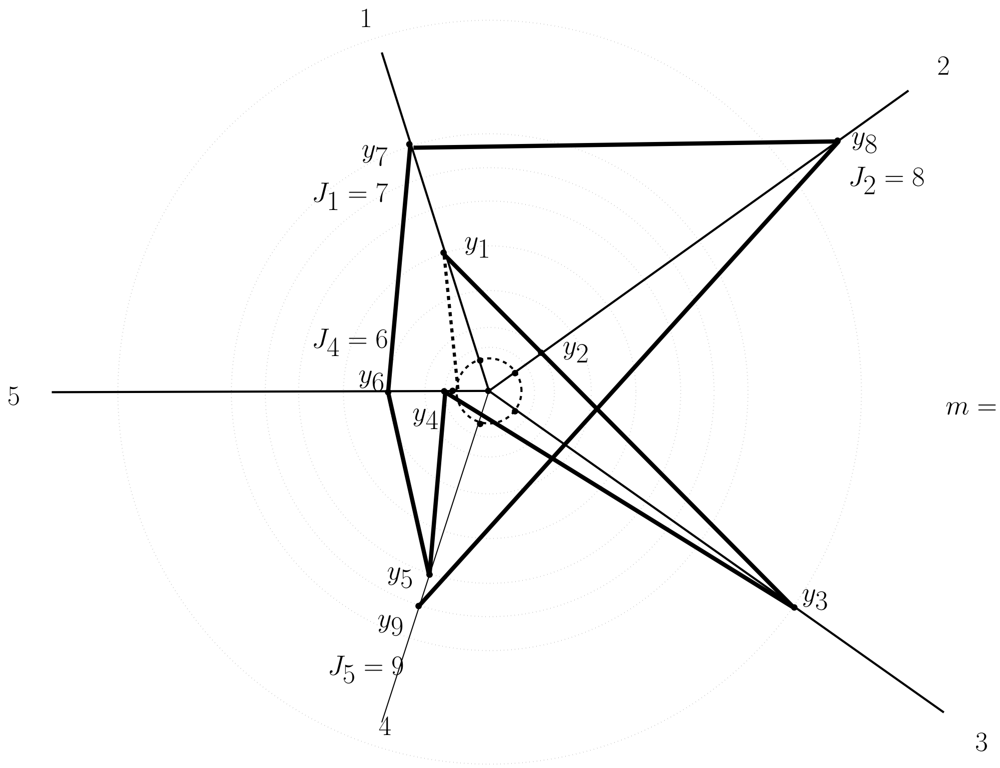
Lower bound construction: m *imaginary* rays

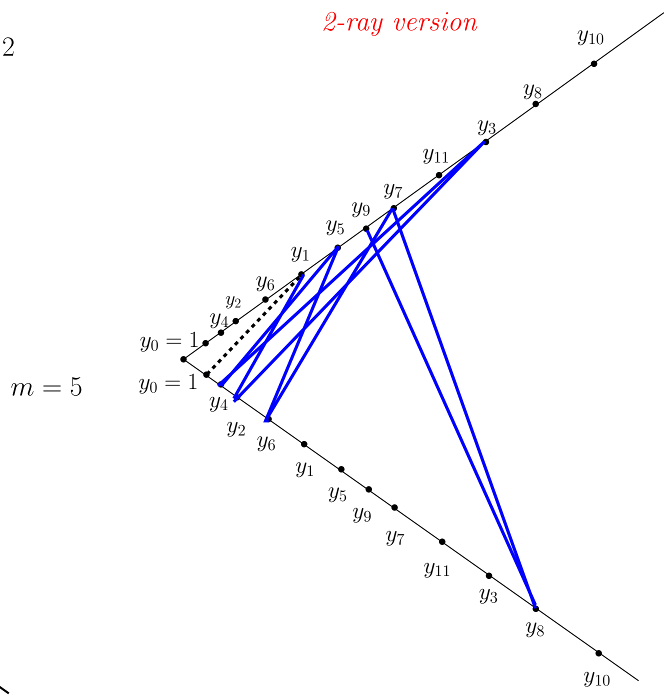
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1}\cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!



Lower bound construction: m *imaginary* rays

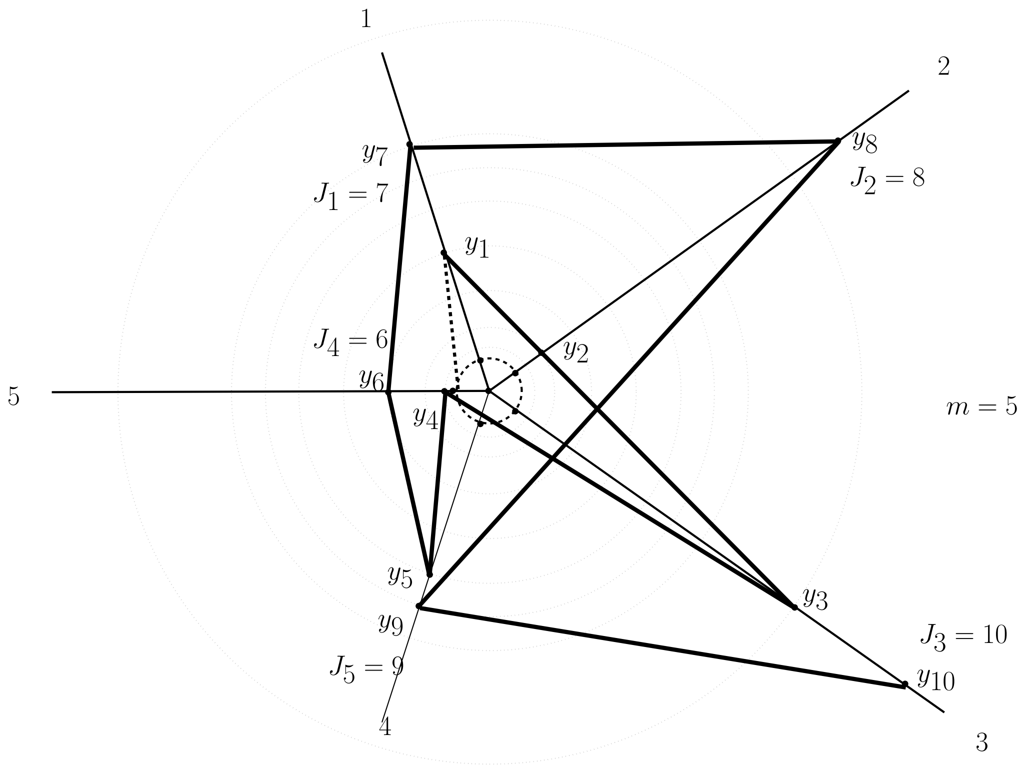
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

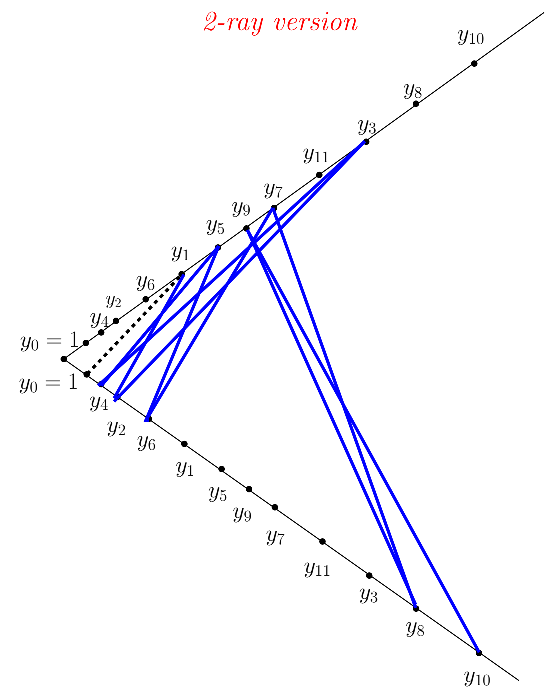




Lower bound construction: m *imaginary* rays

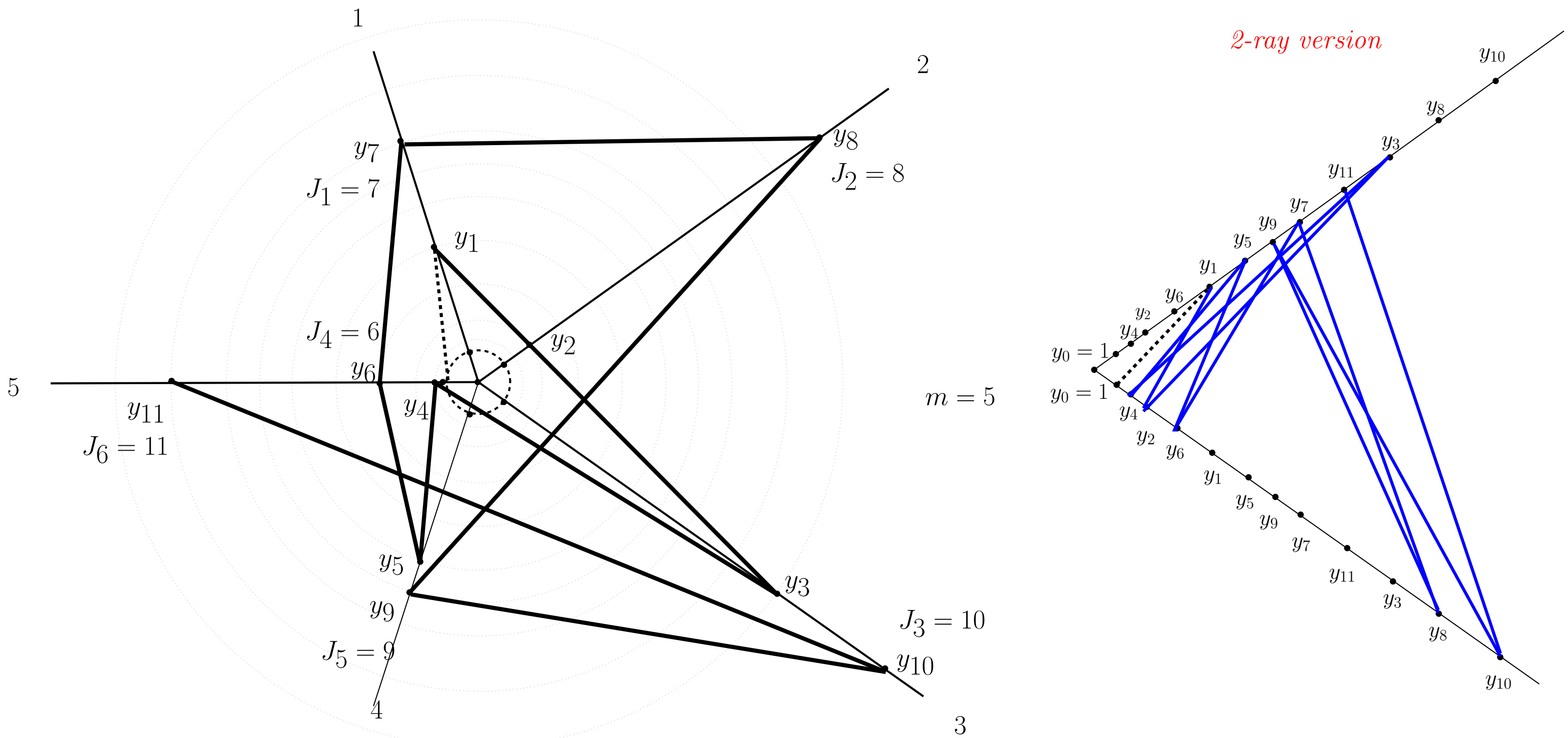
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!





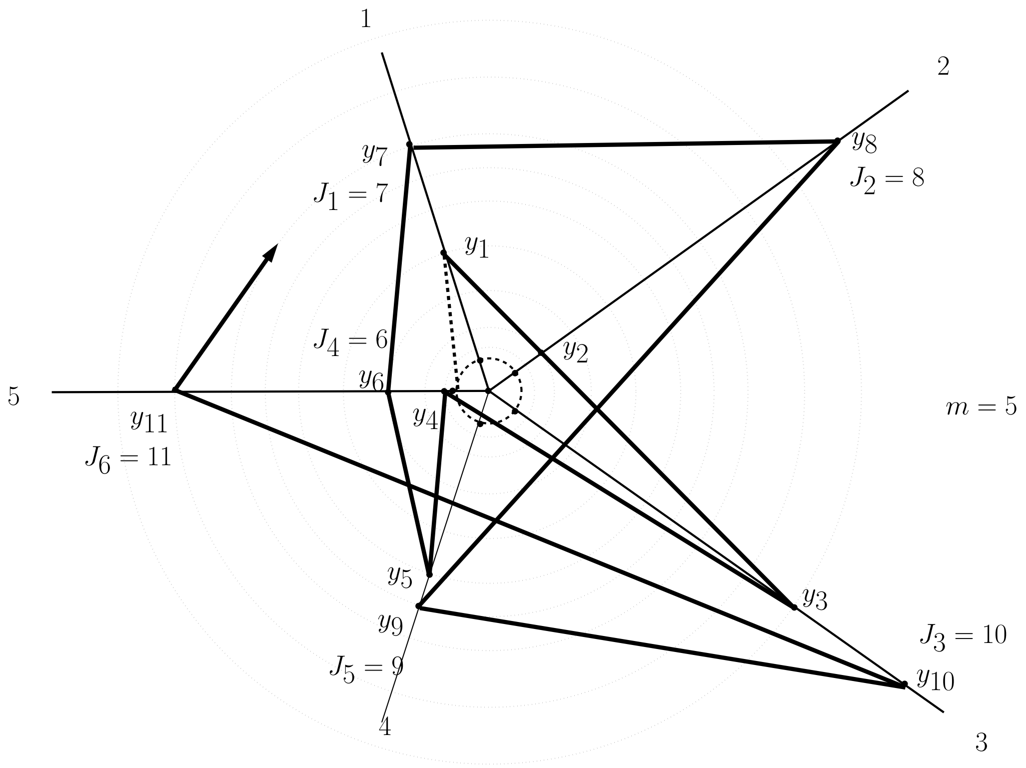
Lower bound construction: m *imaginary* rays

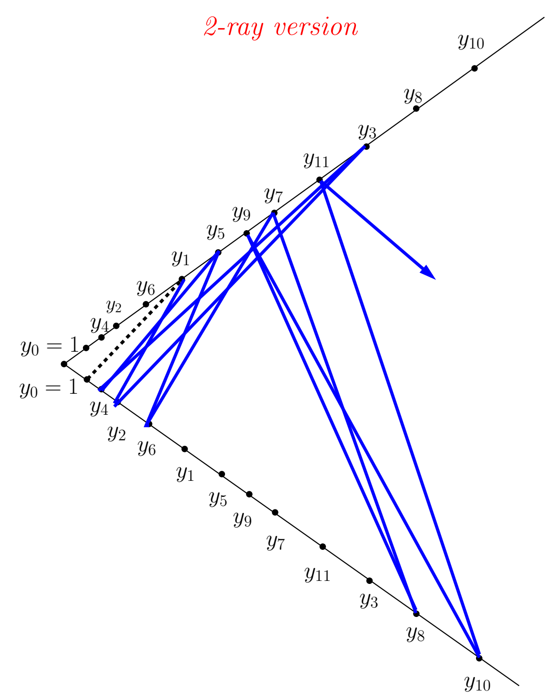
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_iy_{i+1}\cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!



Lower bound construction: m *imaginary* rays

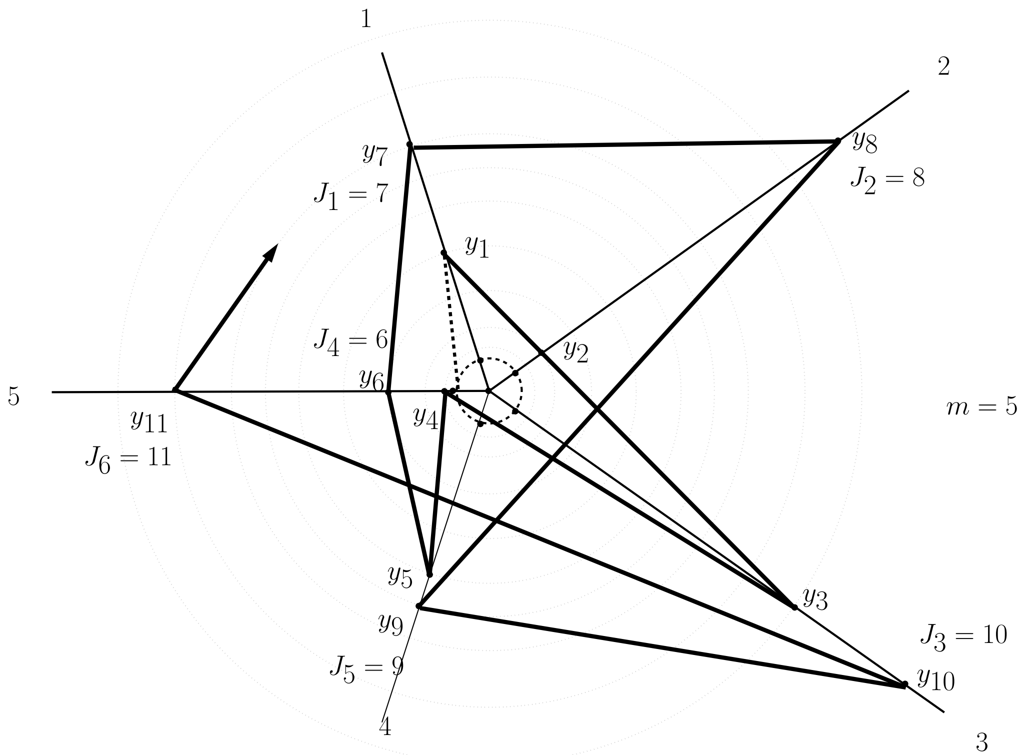
$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_iy_{i+1}\cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!

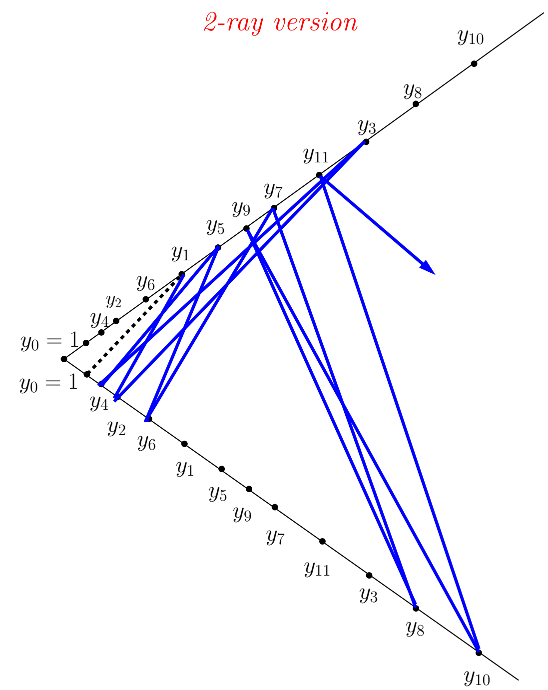




Lower bound construction: m *imaginary* rays

$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, indices J_k original *visiting order*!





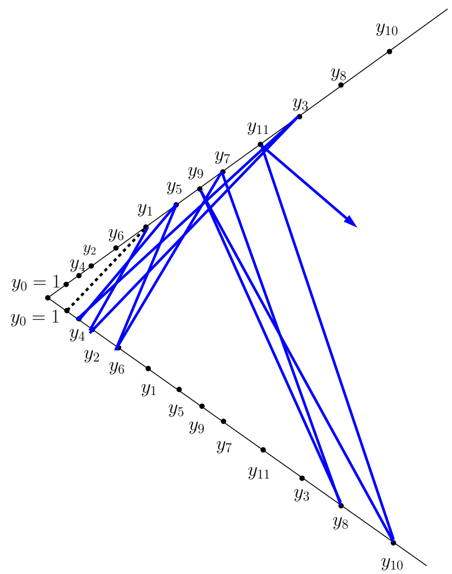
Visiting order: $J_1 = 7, J_2 = 8, J_3 = 10, J_4 = 6, J_5 = 9, J_6 = 11, \ldots$

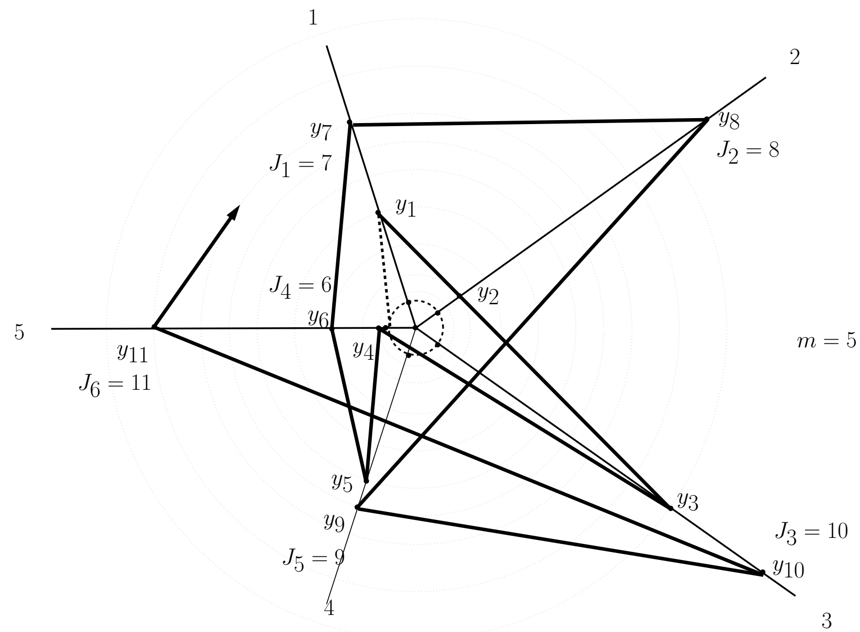
Lower bound construction: m *imaginary* rays

Lower bound construction: m *imaginary* rays

$\sup_k \frac{\sum_{i=1}^{J_k - 2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$ *visiting order:*

$J_1 = 7, J_2 = 8, J_3 = 10, J_4 = 6, J_5 = 9, J_6 = 11, \ldots$



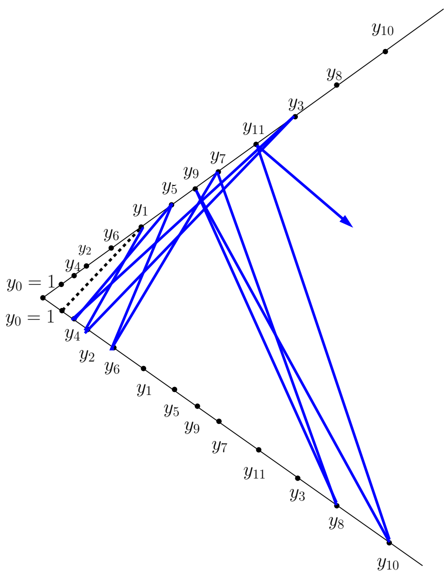


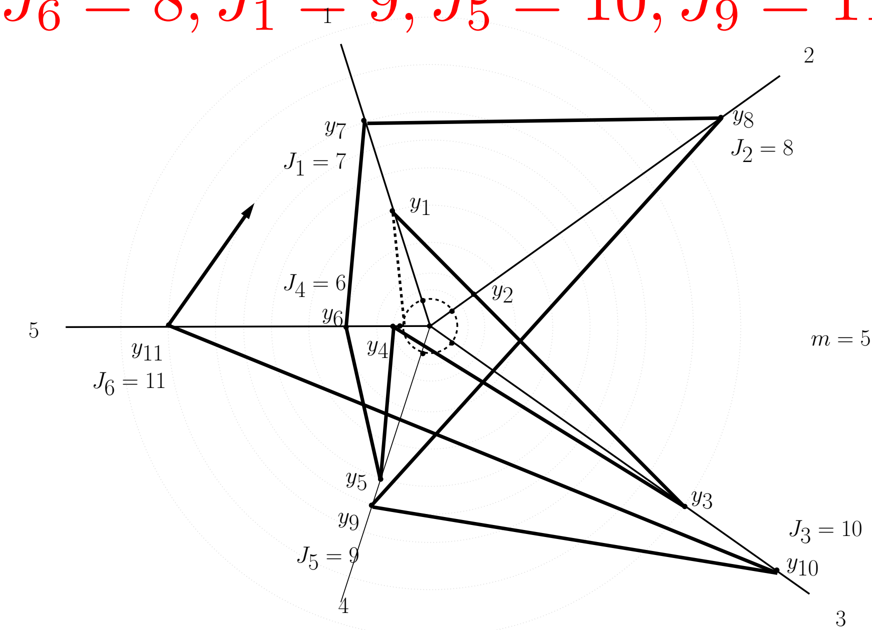
Lower bound construction: m *imaginary* rays

$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$ *visiting order:*

$J_1 = 7, J_2 = 8, J_3 = 10, J_4 = 6, J_5 = 9, J_6 = 11, \ldots$

Optimize: $J_4 = 6, J_2 = 7, J_6 = 8, J_1 = 9, J_5 = 10, J_9 = 11, \ldots$





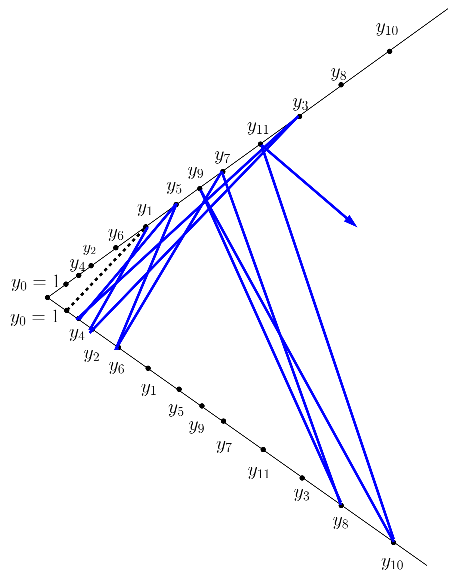
Optimization I: 2-rays and smallest current depth visiting order!!

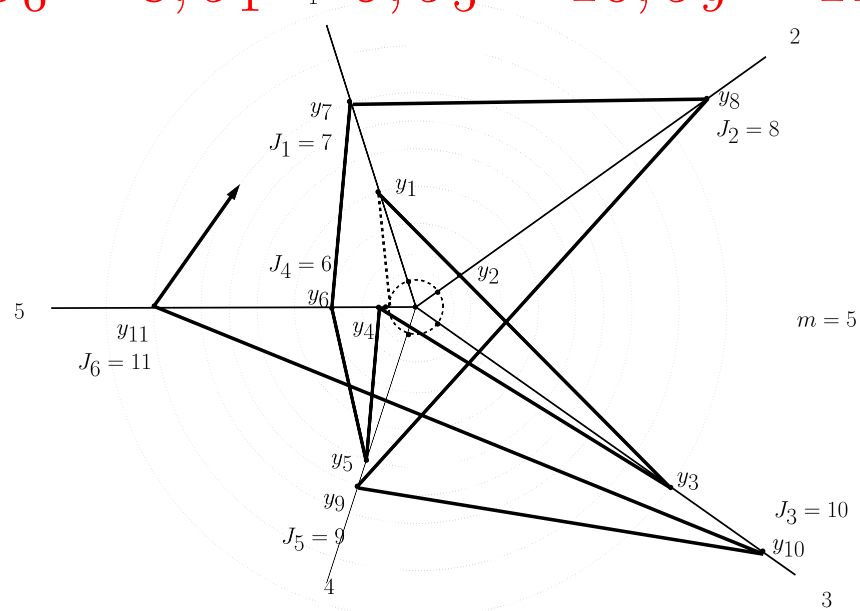
Lower bound construction: m *imaginary* rays

$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$ *visiting order:*

$J_1 = 7, J_2 = 8, J_3 = 10, J_4 = 6, J_5 = 9, J_6 = 11, \ldots$

Optimize: $J_4 = 6, J_2 = 7, J_6 = 8, J_1 = 9, J_5 = 10, J_9 = 11, \ldots$





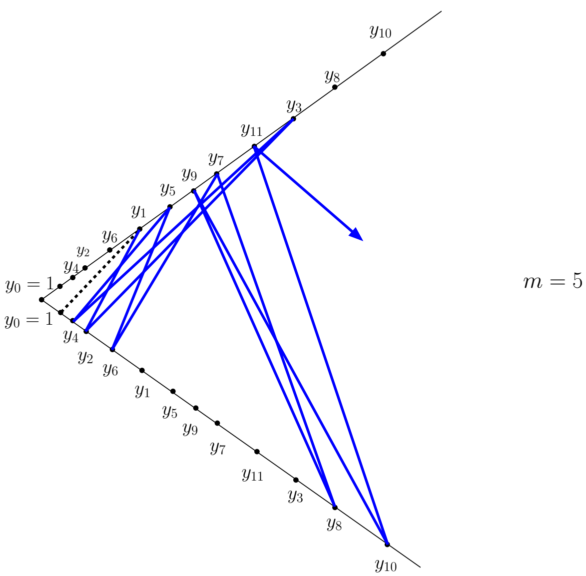
Optimization I: 2-rays and smallest current depth visiting order!!

Lower bound construction: m *imaginary* rays

Lower bound construction: m *imaginary* rays

$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$ *opt. visiting order:*

$J_4 = 6, J_2 = 7, J_6 = 8, J_1 = 9, J_5 = 10, J_9 = 11, \ldots$

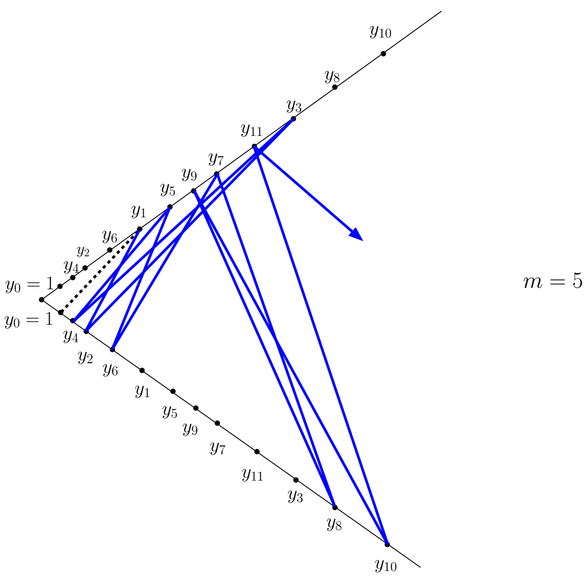


Lower bound construction: m *imaginary* rays

$$\sup_k \frac{\sum_{i=1}^{J_k - 2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$ *opt. visiting order:*

$J_4 = 6, J_2 = 7, J_6 = 8, J_1 = 9, J_5 = 10, J_9 = 11, \ldots$

Optimize numerator by reordering!

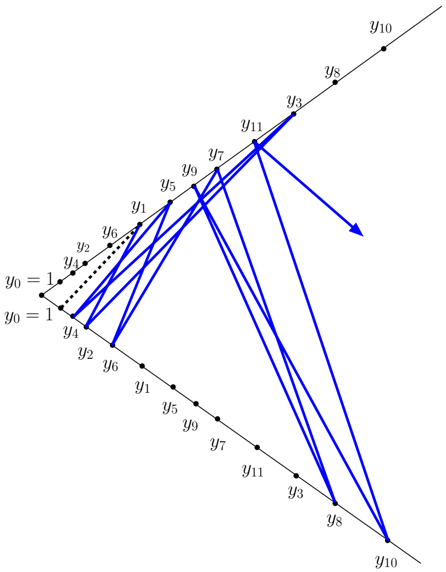


Lower bound construction: m *imaginary* rays

$\sup_k \frac{\sum_{i=1}^{J_k - 2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$ *opt. visiting order:*

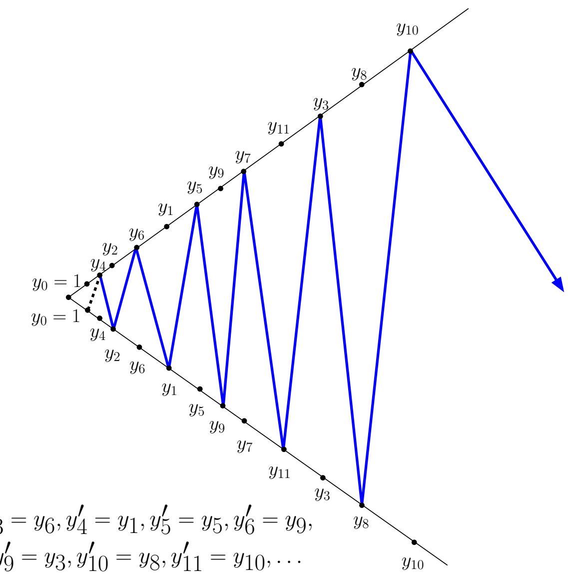
$J_4 = 6, J_2 = 7, J_6 = 8, J_1 = 9, J_5 = 10, J_9 = 11, \ldots$

Optimize numerator by reordering!



$m = 5$

$y'_1 = y_4, y'_2 = y_2, y'_3 = y_6, y'_4 = y_1, y'_5 = y_5, y'_6 = y_9,$
$y'_7 = y_7, y'_8 = y_{11}, y'_9 = y_3, y'_{10} = y_8, y'_{11} = y_{10}, \ldots$

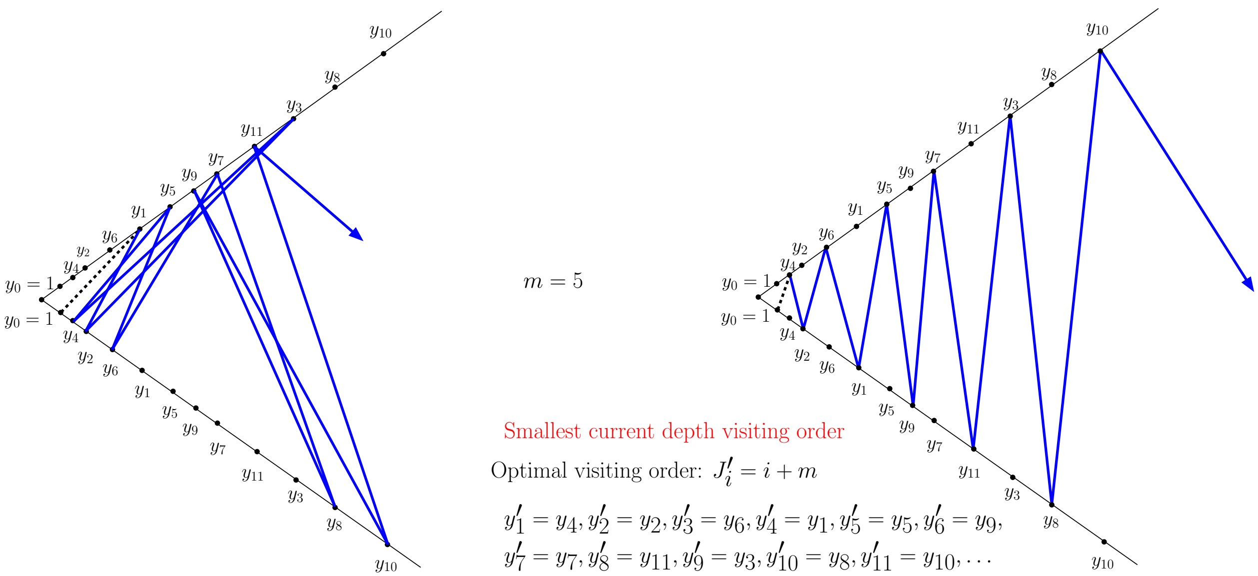


Lower bound construction: m *imaginary* rays

$$\sup_k \frac{\sum_{i=1}^{J_k - 2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$ *opt. visiting order:*

$J_4 = 6, J_2 = 7, J_6 = 8, J_1 = 9, J_5 = 10, J_9 = 11, \ldots$

Optimize numerator by reordering!

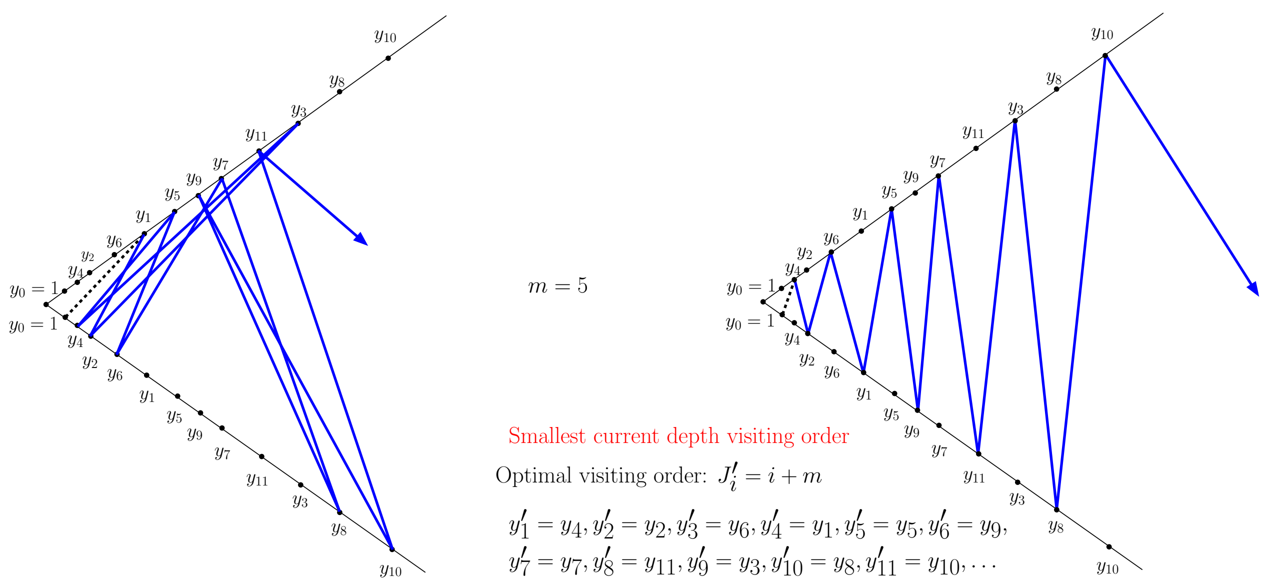


Lower bound construction: m *imaginary* rays

$$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$ *opt. visiting order:*

$J_4 = 6, J_2 = 7, J_6 = 8, J_1 = 9, J_5 = 10, J_9 = 11, \ldots$

Optimize numerator by reordering!

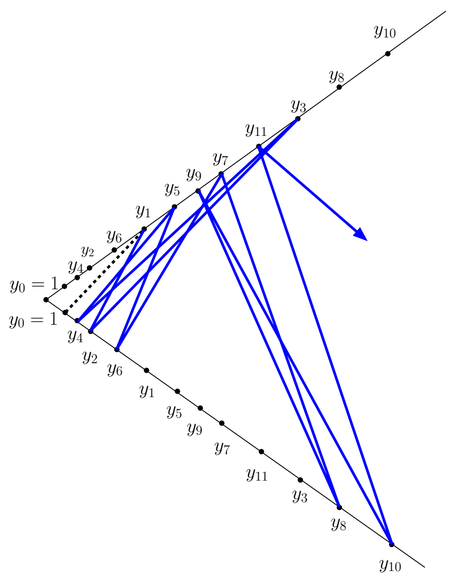


Lower bound construction: m *imaginary* rays

$\sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_iy_{i+1}\cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$ *opt. visiting order:*

$J_4 = 6, J_2 = 7, J_6 = 8, J_1 = 9, J_5 = 10, J_9 = 11, \ldots$

Optimize numerator by reordering!

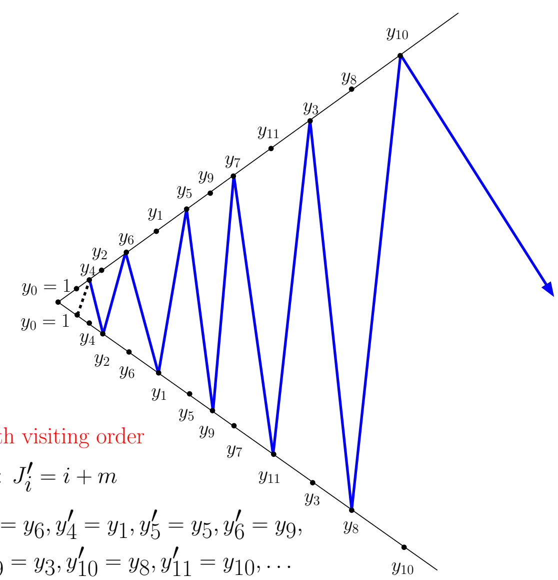


$m = 5$

Smallest current depth visiting order

Optimal visiting order: $J_i' = i + m$

$y_1' = y_4, y_2' = y_2, y_3' = y_6, y_4' = y_1, y_5' = y_5, y_6' = y_9,$
$y_7' = y_7, y_8' = y_{11}, y_9' = y_3, y_{10}' = y_8, y_{11}' = y_{10}, \ldots$



Opt. II: 2-rays, reordering and smallest current depth visiting order!!

Lower bound construction

Lower bound construction

$$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k},$$ J_k original *visiting order*!

Lower bound construction

$$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k - 2} \sqrt{y_i^2 - 2 y_i y_{i+1} \cos \frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2 y'_i y_{i+1} \cos \frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Lower bound construction

$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1}\cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2y'_i y_{i+1}\cos\frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Framework of Gal, discrete m, continous, unimodality etc.

Lower bound construction

$$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2y'_i y_{i+1} \cos\frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Framework of Gal, discrete m, continous, unimodality etc.

Optimized by $y'_i = a^i$, ratio: $f(a,m) = \frac{a^{m-1}}{a-1}\sqrt{1 - 2a\cos\frac{2\pi}{m} + a^2}$

Lower bound construction

$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2y'_i y_{i+1} \cos\frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Framework of Gal, discrete m, continous, unimodality etc.

Optimized by $y'_i = a^i$, ratio: $f(a,m) = \frac{a^{m-1}}{a-1}\sqrt{1 - 2a\cos\frac{2\pi}{m} + a^2}$

$m \mapsto \infty$ then $f(a_{\min}, m) \mapsto 17.289\ldots$

Lower bound construction

$C(S) \geq \sup_k \frac{\sum_{i=1}^{J_k-2} \sqrt{y_i^2 - 2y_i y_{i+1} \cos\frac{2\pi}{m} + y_{i+1}^2}}{y_k}$, J_k original *visiting order*!

Minimized by $\sup_n \frac{\sum_{i=1}^{n+m-2} \sqrt{{y'_i}^2 - 2y'_i y_{i+1} \cos\frac{2\pi}{m} + {y'_{i+1}}^2}}{y'_n}$

Framework of Gal, discrete m, continous, unimodality etc.

Optimized by $y'_i = a^i$, ratio: $f(a,m) = \frac{a^{m-1}}{a-1}\sqrt{1 - 2a\cos\frac{2\pi}{m} + a^2}$

$m \mapsto \infty$ then $f(a_{\min}, m) \mapsto 17.289\ldots$

Example: $m = 100000$, compute $a_{\min} = 1.000009764\ldots$ and $f(a_{\min}, 100000) = 17.289\ldots$

Conclusion

Conclusion

- Searching for a point in the plane

Conclusion

- Searching for a point in the plane
- Spiral search is optimal

Conclusion

- Searching for a point in the plane
- Spiral search is optimal
- Smallest current depth visiting order: powerful tool

Conclusion

- Searching for a point in the plane
- Spiral search is optimal
- Smallest current depth visiting order: powerful tool
- Applicable to standard m ray search

Conclusion

- Searching for a point in the plane
- Spiral search is optimal
- Smallest current depth visiting order: powerful tool
- Applicable to standard m ray search
- Spiral conjecture of similar settings