

Online escape paths

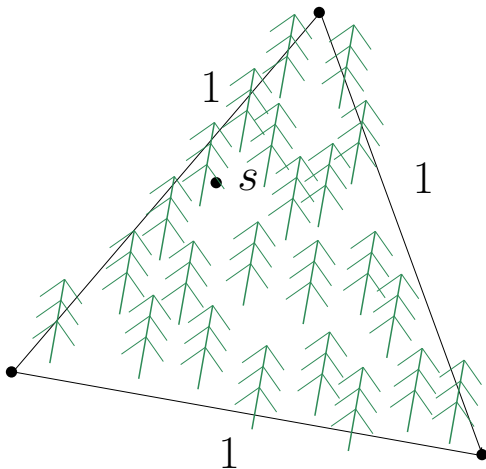
joint work with David Kübel

Elmar Langetepe

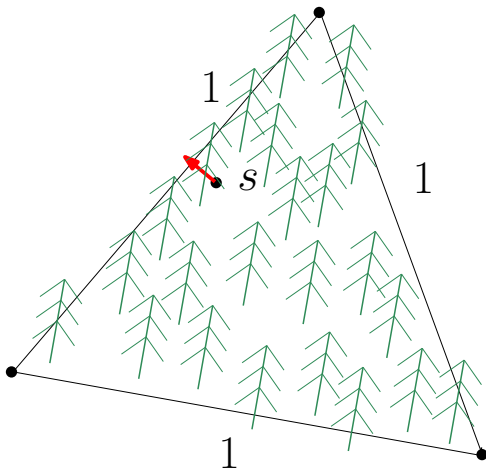
University of Bonn

SWAT 16, June 24th, 2016

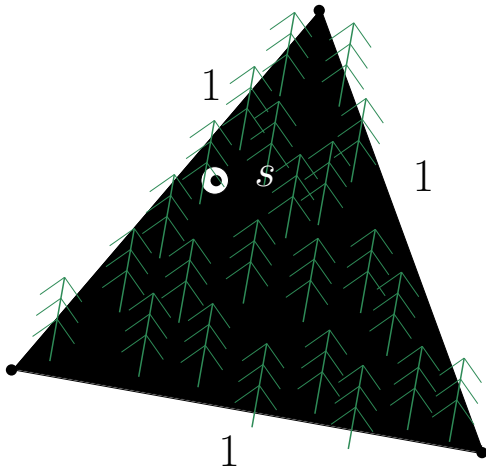
Lost in a forest: Ultimate escape path



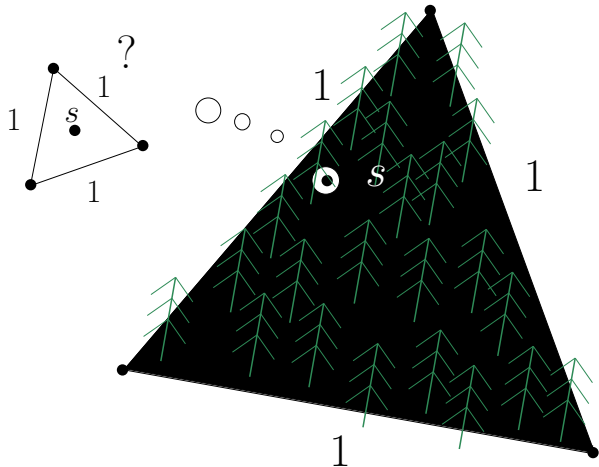
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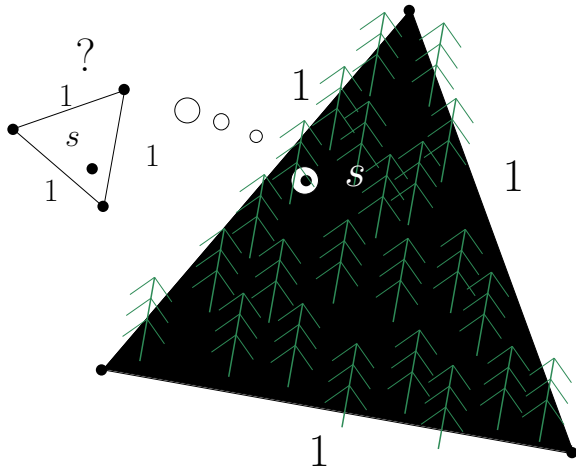
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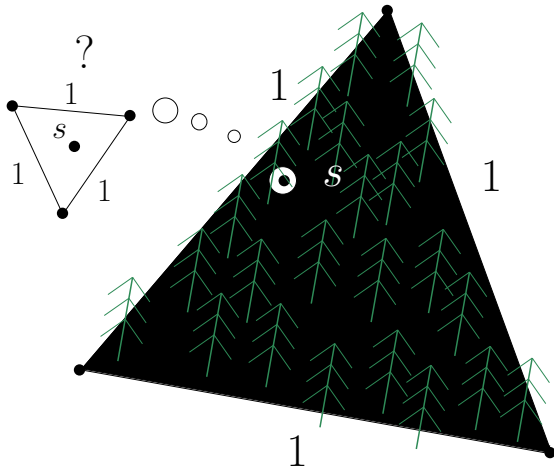
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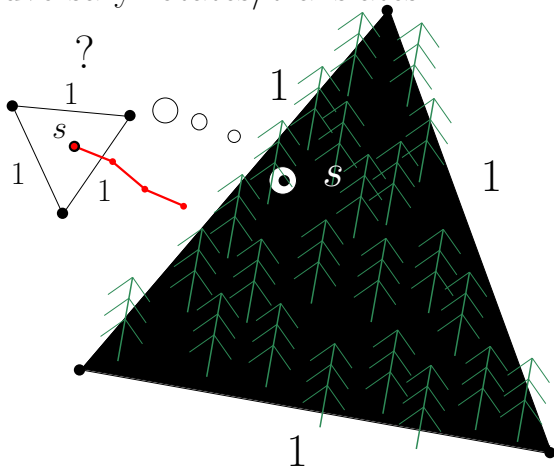


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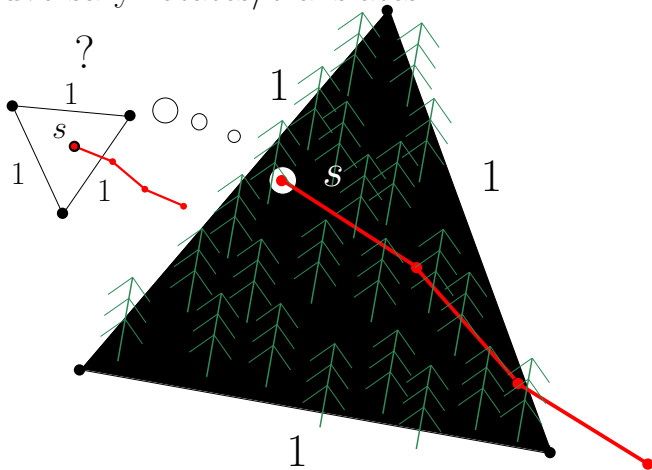
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Adversary rotates/translates



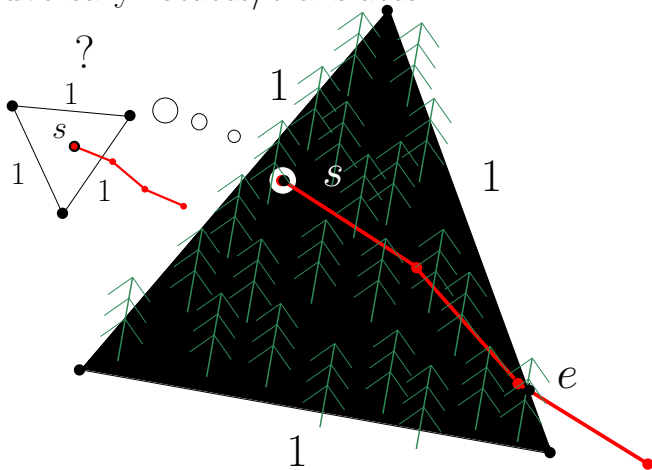
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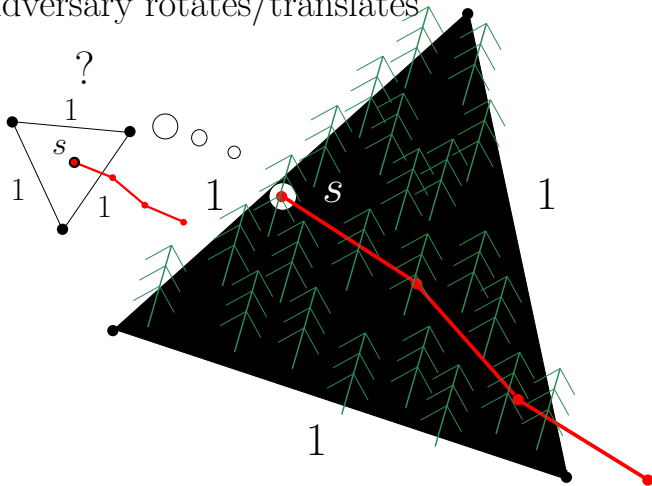
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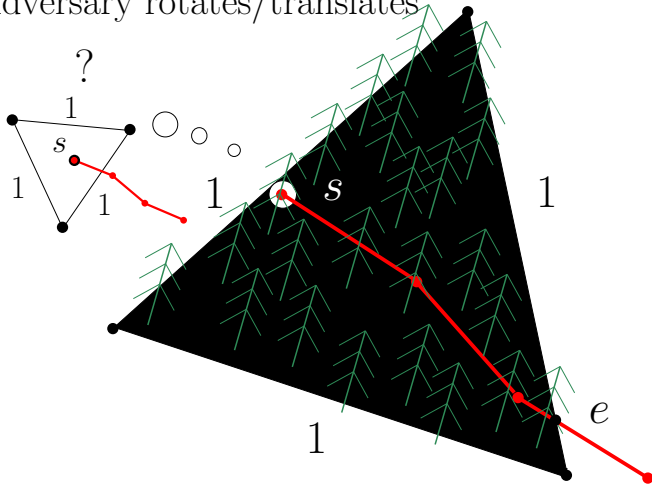
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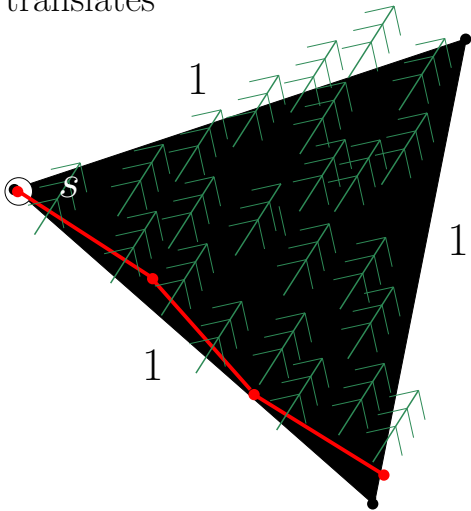
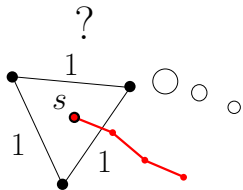
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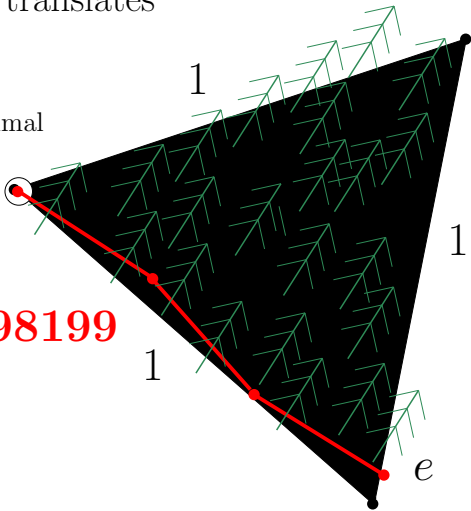


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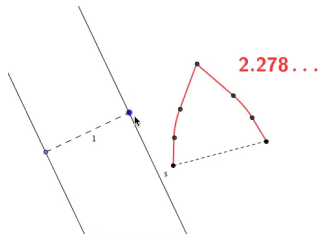
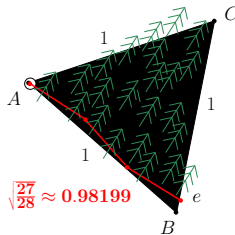
Zig-Zag path is optimal

$$\sqrt{\frac{27}{28}} \approx 0.98199$$



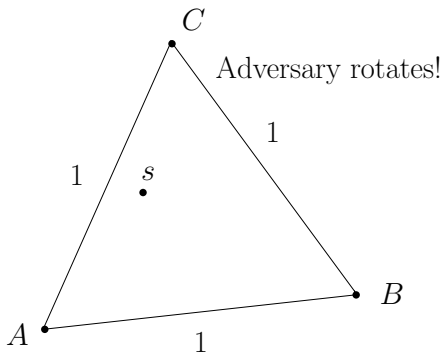
Lost in a forest: Ultimate escape path

- Bellman (1956):
Lost in a forest
- Besicovitch (1965):
Zig-Zag-Path
- Coulton/Movshovich (2006):
Optimality
- Very few examples
Strip/Zalgaller



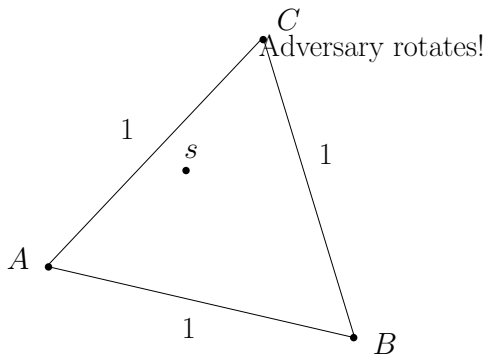
Lost in a forest: Certificate path

- Ultimate escape path only known for some convex shapes
- I. Adversary only rotates! II. Simple escape path!



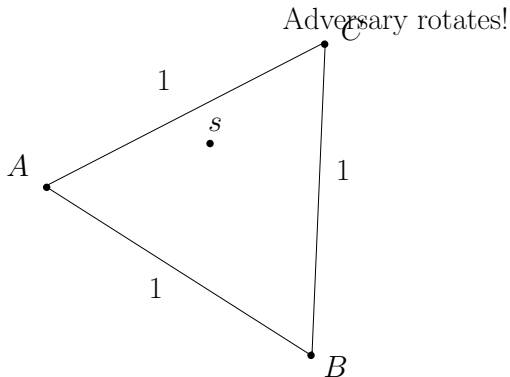
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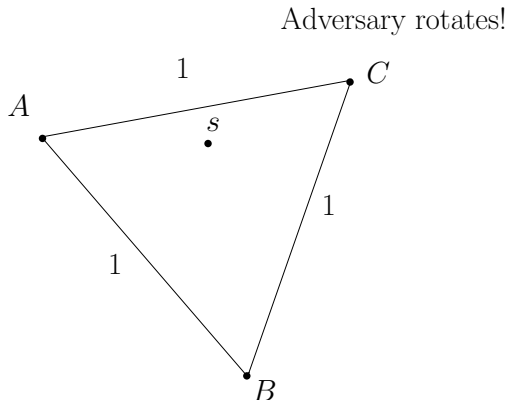
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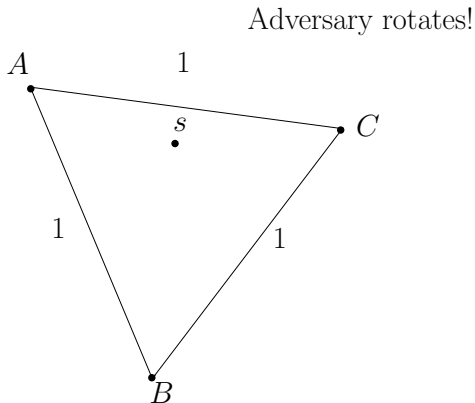
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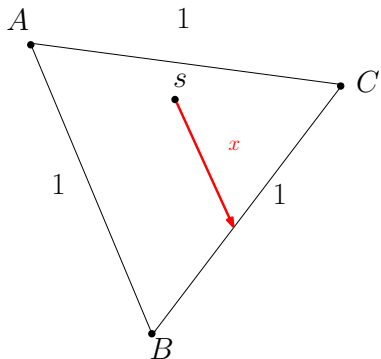
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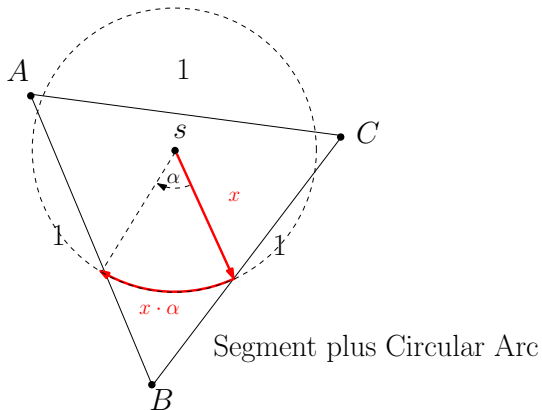
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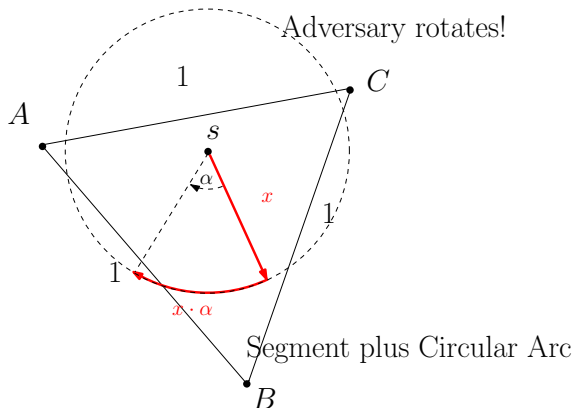
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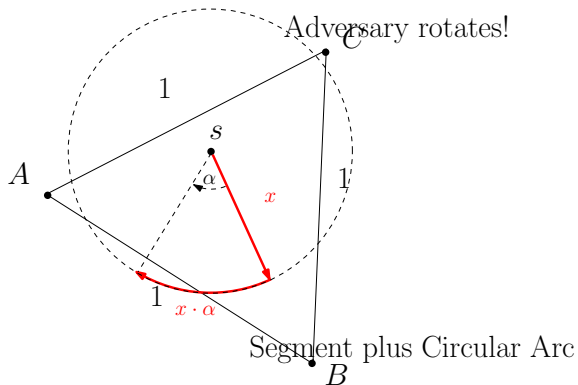
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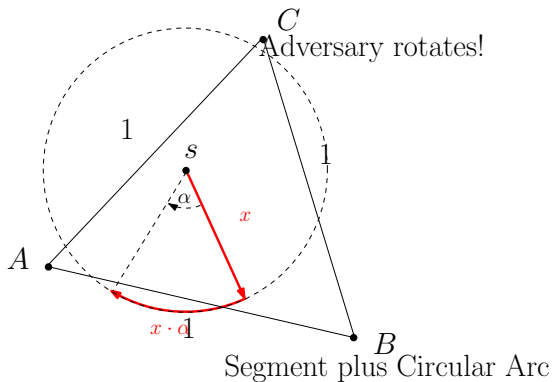
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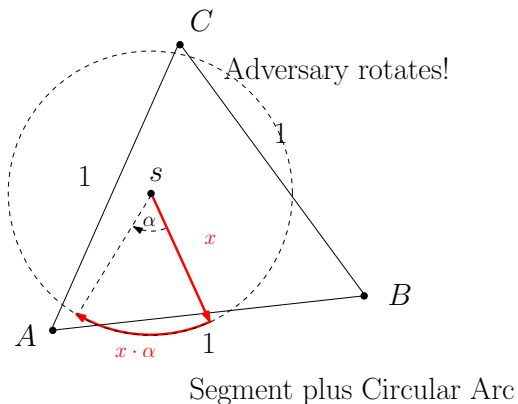
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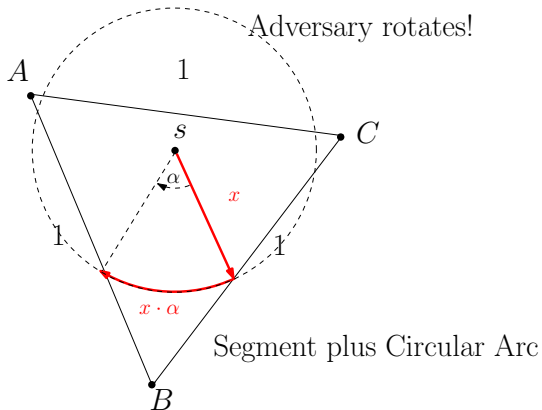
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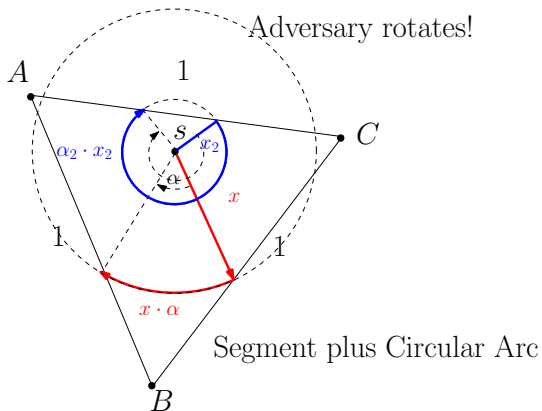
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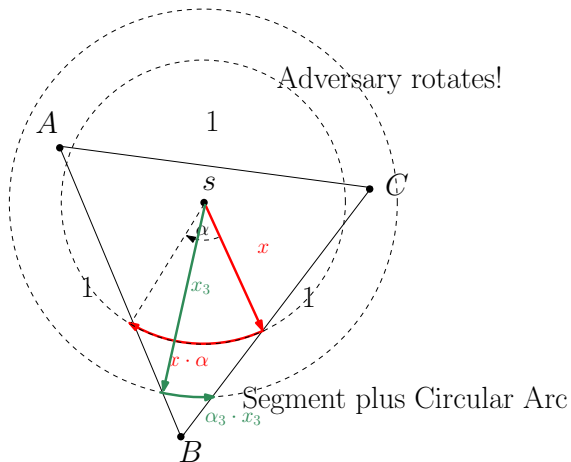
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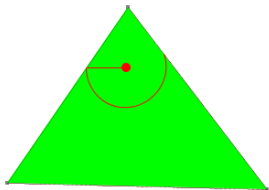
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Shortest segment/arc combination not covered by rotation!

Advantage: Certificate vs. Ultimate

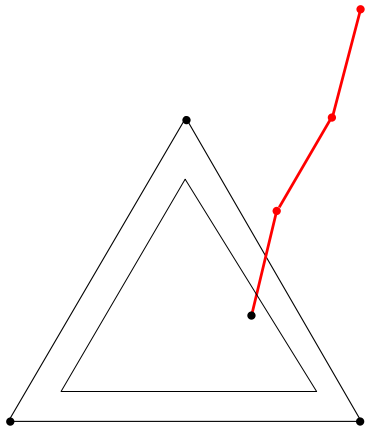
- Easy to compute, more general environments
- Beats ultimate path, more information



- Polynomial time
- Non-convex
- Star-shaped

Advantage: Certificate vs. Ultimate

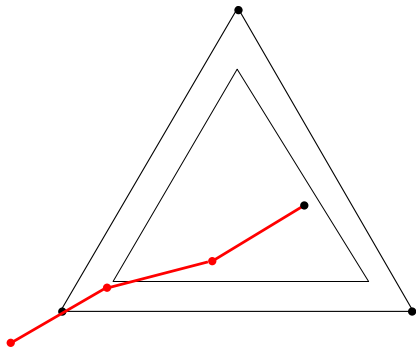
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- Triangle
- Zig-Zag path
- Certificate
- Better candidates!

Advantage: Certificate vs. Ultimate

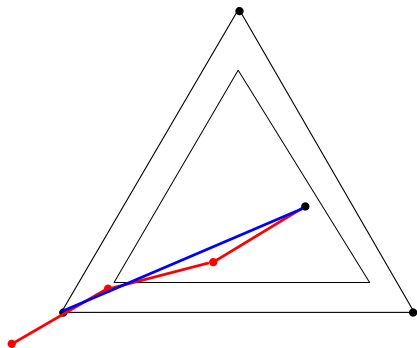
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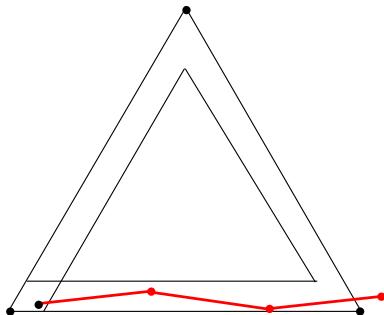
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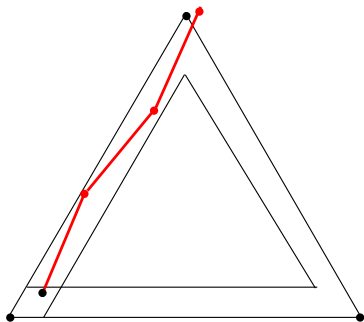
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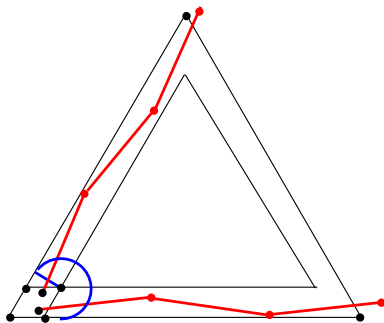
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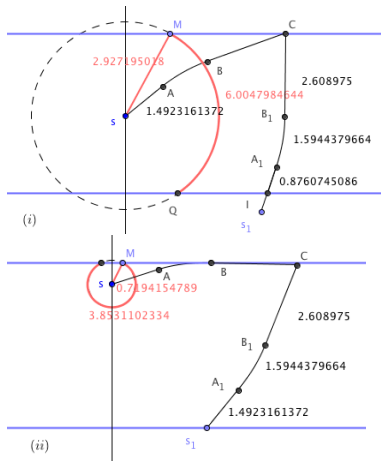
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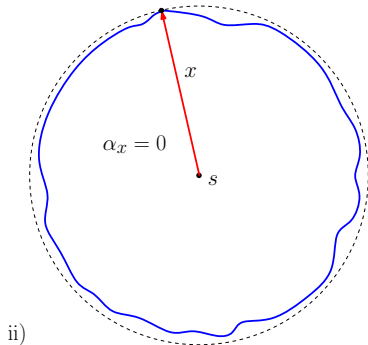
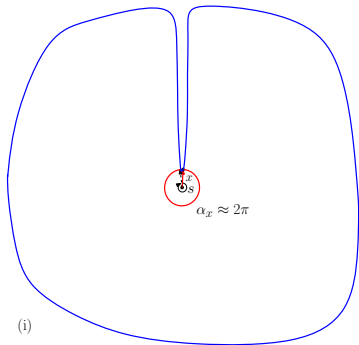
- Easy to compute, more general environments
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- Strip
- Zalgaller path
- Certificate
- Better candidates!

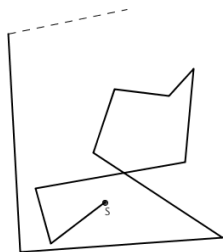
Certificate: Extreme Cases!

- Few small distances: $\approx x(1 + 2\pi)$
- All distances roughly the same: $\approx x(1 + 0)$
- Certificate $\approx x(1 + \alpha_x)$ with $\alpha_x \in [0, 2\pi]$!



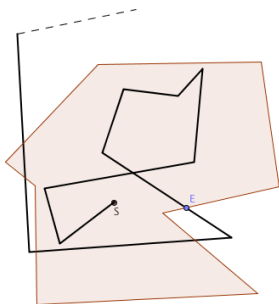
Another turn of the screw: Online!

- NO sight, NO knowledge of shape AND position!
- Online escape path! Unknown certificate! Spiral strategy!
- Logarithmic spiral vs. certificates! Cover unknown certificate!



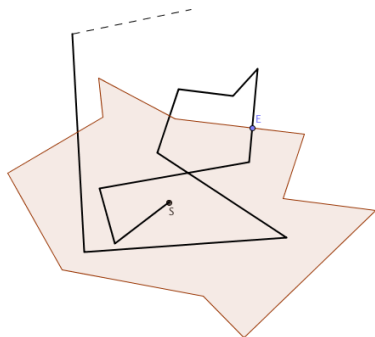
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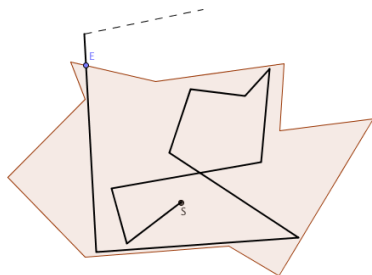
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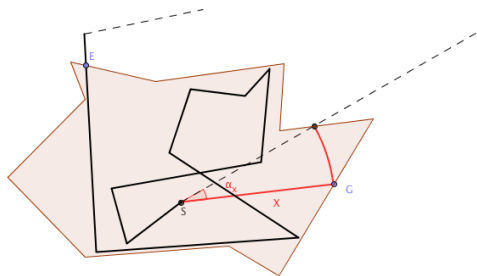
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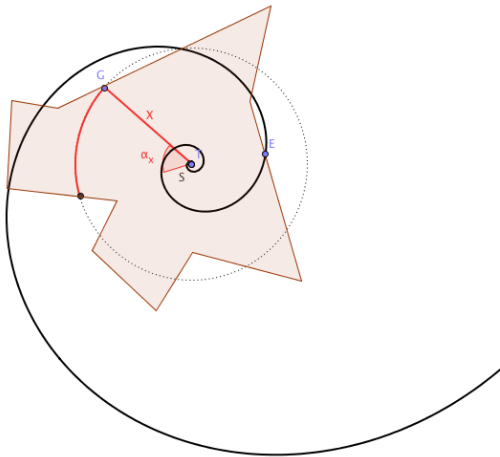
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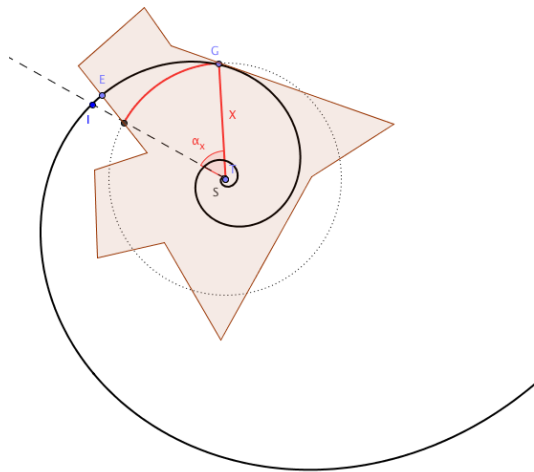
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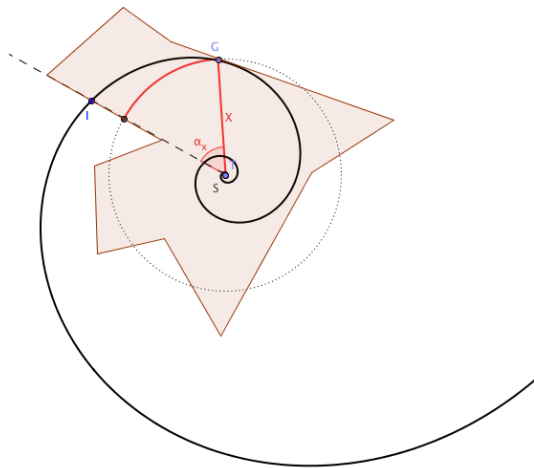
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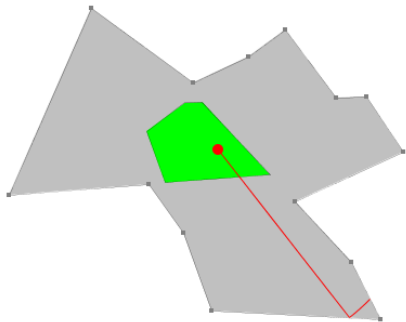
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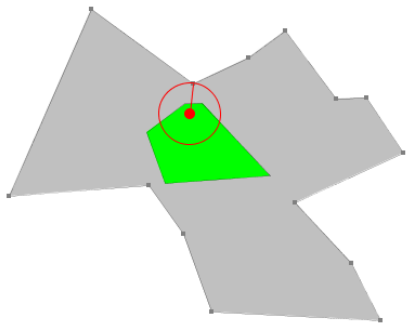
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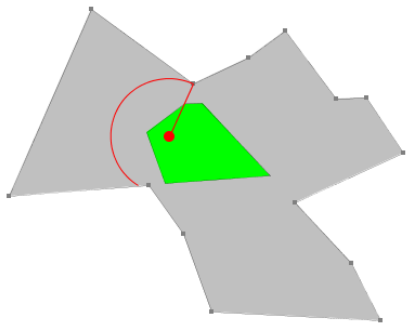
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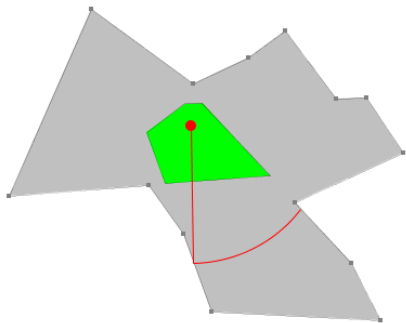
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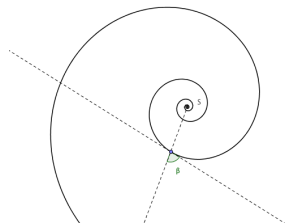
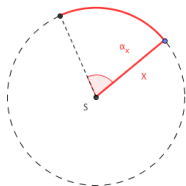
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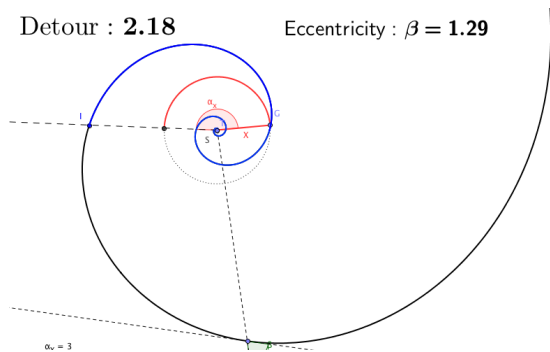
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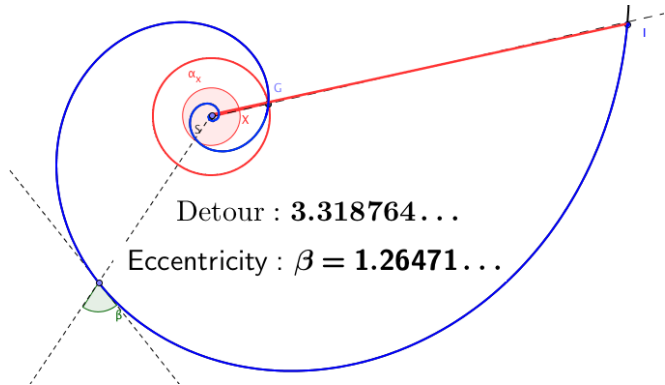
Online spiral path against arbitrary certificate

- Online spiral escape path with eccentricity β !
- Detour against arbitrary certificate: Competitive ratio!

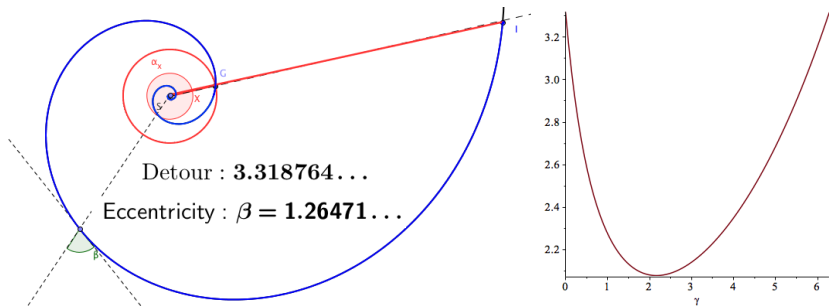


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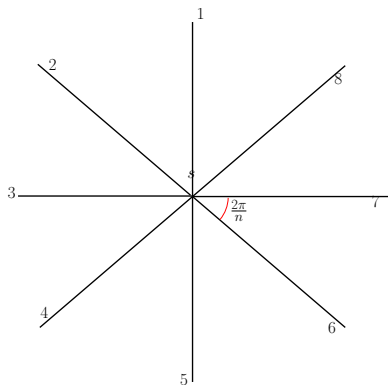
Online spiral path against arbitrary certificate



- The same worst-case ratio for both extremes! Analytically!
- **Theorem I:** There is a spiral strategy that escape from *any unknown environment* (kernel/star-shaped) and has detour at most $3.318764\dots$ against the corresponding certificate path for the known shape.

Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

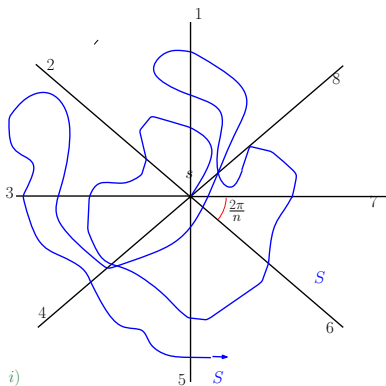


Detour at least $D/2 \geq 3.313126!$

n goes to infinity!

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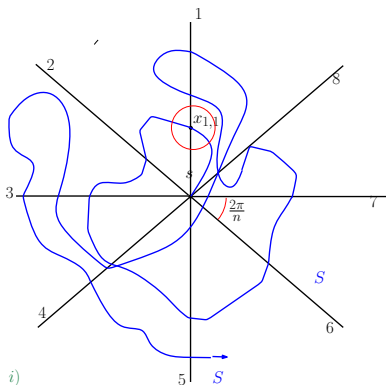


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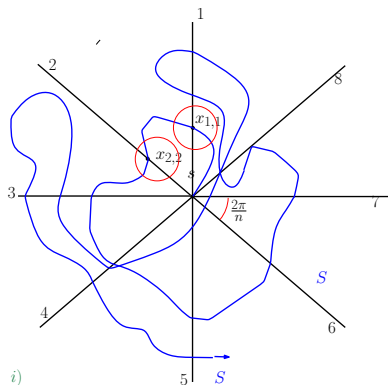


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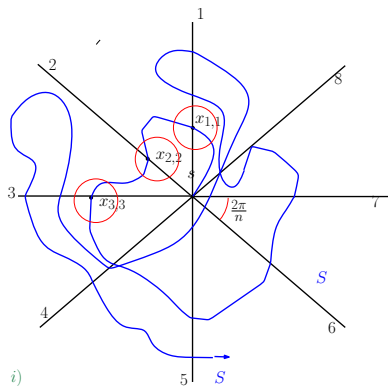


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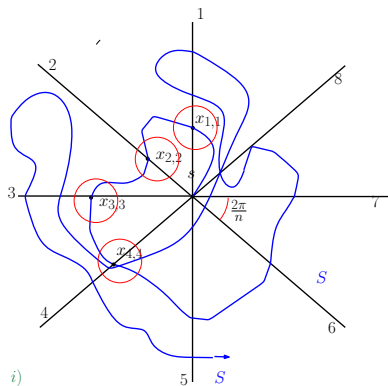


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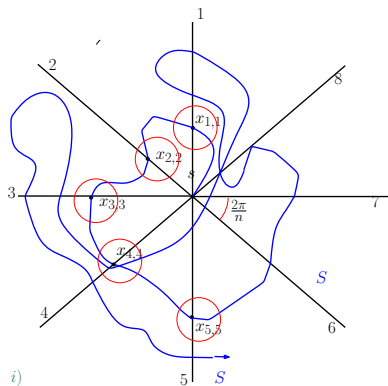


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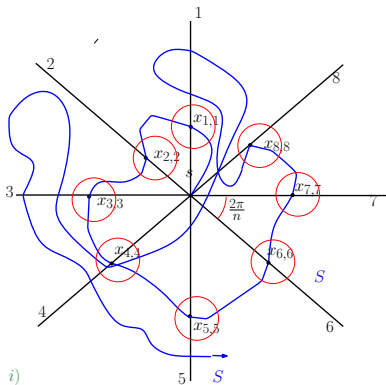


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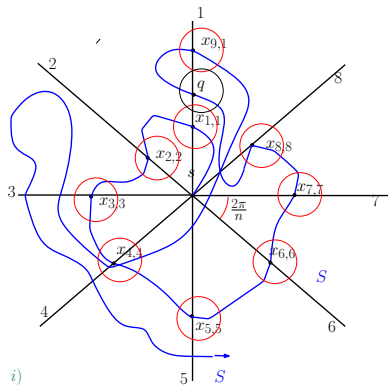


Detour at least $D/2 \geq 3.313126!$

n goes to infinity!

Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

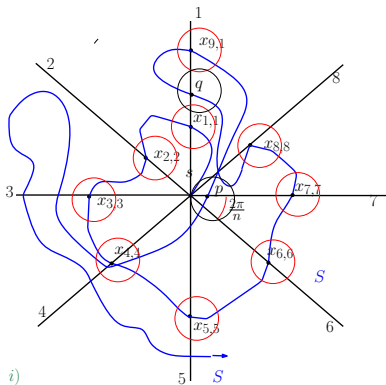


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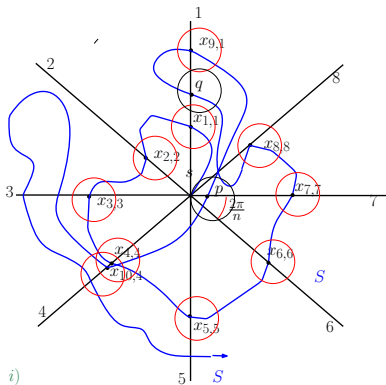


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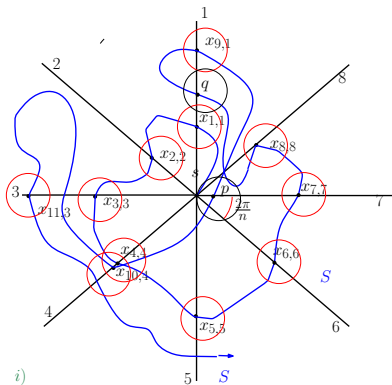


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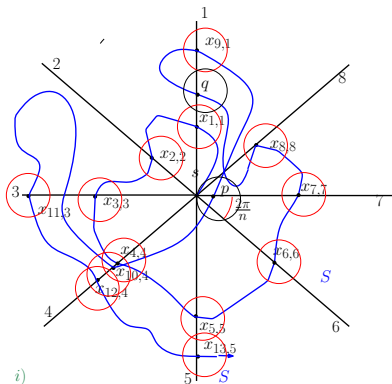


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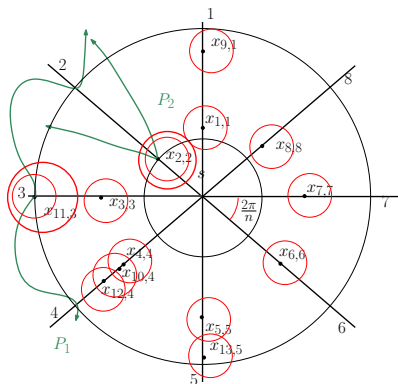


Detour at least $D/2 \geq 3.313126!$

n goes to infinity!

Online spiral strategy is almost optimal!

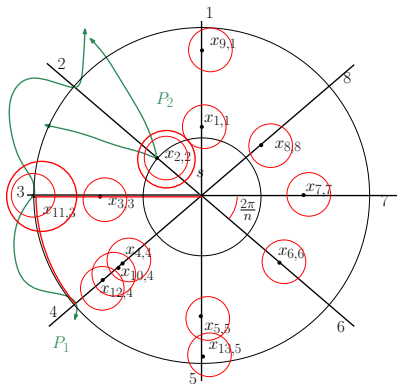
- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!



Detour at least $D/2 \geq 3.313126!$
 n goes to infinity!

Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
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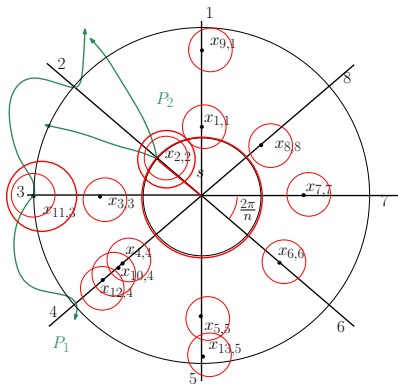


Detour at least $D/2 \geq 3.313126!$

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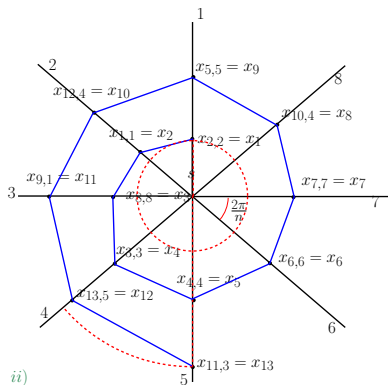


Detour at least $D/2 \geq 3.313126!$

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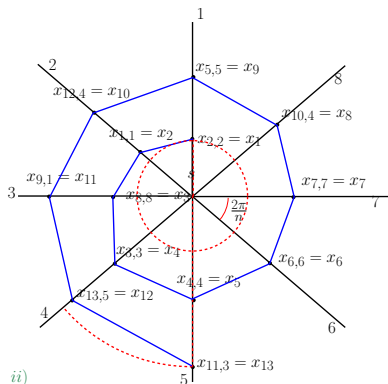


Detour at least $D/2 \geq 3.313126!$

n goes to infinity!

Online spiral strategy is almost optimal!

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- Arbitrary strategy: Discretization, Reordering, Functionals!



Lower bound: Minimize

$$\frac{\sum_{i=1}^{k-1} \sqrt{x_i^2 - 2 \cos\left(\frac{2\pi}{n}\right) x_i x_{i+1} + x_{i+1}^2}}{x_k \left(1 + \frac{2\pi}{n}\right)} +$$

$$\frac{\sum_{i=1}^{k-1} \sqrt{x_i^2 - 2 \cos\left(\frac{2\pi}{n}\right) x_i x_{i+1} + x_{i+1}^2}}{x_{k-n+1} (1 + 2\pi)}$$

for all n and k .

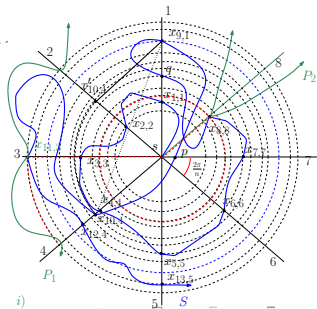
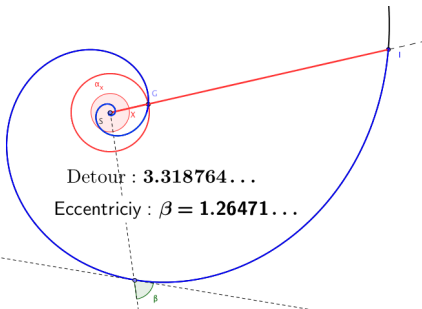
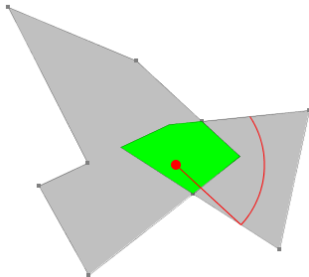
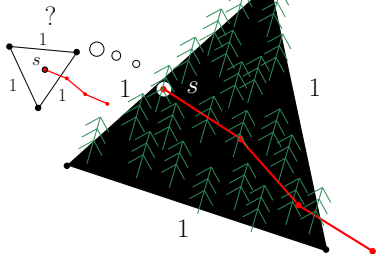
Gives at least $D = 6.62521\dots$ for the sum of Detours!

Detour at least $D/2 \geq 3.313126!$

n goes to infinity!

Summary!

Adversary rotates/translates



Theorem: Escape from an unknown environment against the best certificate can be done within a competitive ratio of $3.318764\dots$ and this is (almost ≈ 0.005) tight.